Conference on Mathematics of Wave Phenomena

February 24-28, 2025

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Book of Abstracts

Conference on Mathematics of Wave Phenomena 2025

held at the Department of Mathematics, Karlsruhe Institute of Technology, Germany, February 24–28, 2025

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wave phenomena
analysis and numerics

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Preface

The mathematical modeling, simulation and analysis of wave phenomena entail a plethora of fascinating and challenging problems both in analysis and numerical mathematics. During the past decades, these challenges have inspired a number of important approaches, developments and results about wave-type equations in both fields of mathematics. As the previous editions of 2018 and 2022, our

Conference on Mathematics of Wave Phenomena 2025

brings together international experts with different background to stimulate the transfer of ideas, results, and techniques within this exciting area.

Eleven plenary lectures by world leading experts will provide detailed insights in various topics in the field. Many colleagues have contributed by organizing one of twenty-three minisymposia that cover a wide range of subjects within analysis, numerics and applications. These include various aspects of nonlinear waves, dispersion, stability and pattern formation, transient and time-harmonic problems, water waves, acoustics and electromagnetism, waveguides, periodic structures, inverse problems and scattering, multi-scale problems and boundary integral equations. About thirty contributed talks complete our scientific program.

The conference is organized at Karlsruhe Institute of Technology by members of the Collaborative Research Centre 1173 (CRC 1173) on Wave Phenomena. We gratefully acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) through CRC 1173.

Karlsruhe, January 2025

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Plenary Talk Monday, 9:10–9:55, Tulla LH

Forward and Inverse Problems in Nonlinear Acoustics

Barbara Kaltenbacher[1,*]

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The importance of ultrasound is well established in the imaging of human tissue. In order to enhance image quality by exploiting nonlinear effects, recently techniques such as harmonic imaging and nonlinearity parameter tomography have been put forward. As soon as the pressure amplitude exceeds a certain bound, the classical linear wave equation loses its validity and more general nonlinear versions have to be used. Another characteristic property of ultrasound propagating in human tissue is frequency power law attenuation leading to fractional derivative damping models in time domain.

In this talk we will first of all dwell on modeling of nonlinearity on one hand and of fractional damping on the other hand. Then we will give an idea on the challenges in the analysis of the resulting PDEs and discuss some parameter asymptotics. The goal here is to justify some advanced models such as the Jordan-Moore-Gibson-Thompson equation containing a relaxation time, the Blackstock-Crighton equation containing a thermal condictivity parameter, or time-fractional versions of JMGT, by proving that their solutions tend to those of the classical models (the Westervelt equation or Kuznetsov's equation) in appropriate function spaces. This is joint work with Mostafa Meliani (Czech Academy of Sciences), Vanja Nikolić (Radboud University), and Mechthild Thalhammer (University of Innsbruck).

In the second part of the talk, we address some relevant inverse problems in this context, in particular the above mentioned task of nonlinearity parameter imaging, which leads to a coefficient identification problem for a quasilinear wave equation. In particular, we investigate the recovery of the nonlinearity coefficient commonly labeled as B/A in the literature, which is part of a space dependent coefficient κ in the Westervelt equation governing nonlinear acoustics. Corresponding to the typical measurement setup, the overposed data consists of time trace measurements on some manifold Σ representing the receiving transducer array. We will dwell on topics like modeling, uniqueness, numerical reconstruction schemes (in particular based on Newton type methods) as well as simultaneous reconstruction of κ and the sound speed. We will also show some recent results pertaining to the formulation of this problem in frequency domain and numerical reconstruction of piecewise constant coefficients in two space dimensions. This is joint work with Bill Rundell (Texas A&M University).

Plenary Talk **Monday, 10:10–10:55, Tulla LH**

The cubic nonlinear Schrödinger equation on the 2d torus

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This survey talk will be about the Cauchy problem for nonlinear Schrödinger equations on tori $\mathbb{T}^d$, with a focus on the two-dimensional cubic NLS. In this compact setting free solutions do not decay and dispersive effects are more subtle. In the 90's Bourgain established Strichartz-type estimates by means of analytic number theory. More recently, the Fourier decoupling result of Bourgain–Demeter provided a more robust approach which has significantly extended the range of available Strichartz estimates on rational and irrational tori.

In two dimensions the cubic NLS is the L^2-critical problem, which is of particular interest. On the one hand, Bourgain proved local well-posedness in $H^s(\mathbb{T}^2)$ for $s > 0$ and therefore global well-posedness in the energy space $H^1(\mathbb{T}^2)$ under some smallness condition. On the other hand, Colliander-Keel-Staffilani-Takaoka-Tao constructed solutions which transfer energy from small to high frequencies which implies growth of higher Sobolev norms. In $L^2(\mathbb{T}^2)$ the problem cannot be solved perturbatively, which is related to the failure of the scale-invariant L^4 Strichartz estimate.

In joint work with Beomjong Kwak we have found a new method to prove a sharp L^4 estimate by counting parallelograms via the Szemerédi-Trotter theorem. This has led to a new global well-posedness theory for the L^2 critical problem. In this talk, I will describe the new approach and, in particular, I will explain how parallelograms arise in the study of the NLS. The arguments rely on elementary Fourier analysis and incidence geometry.

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Plenary Talk **Tuesday, 9:00–9:45, Tulla LH**

Mathematical models and results for time domain electromagnetic wave propagation in dispersive media

Patrick Joly[1,*], Maxence Cassier[2]
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According to physicists, the dispersion of waves, i.e. the dependence of their speed of propagation w.r.t. the wavelength, is ubiquitous in electromagnetism. The corresponding mathematical models have known a recent gain of interest due to the emergence of metamaterials (manufactured dispersive materials). In my presentation, I intend to present an overview of these models in the time domain (by opposition to the frequency domain) and to present various related results obtained along the past ten years with my collaborators.

In a first part devoted to homogeneous media, I wish to enlighten the role of complex analysis, in particular the theory of Herglotz functions, in the notion of passive models, with an emphasis on so-called Generalized Lorentz models. In the case of non dissipative media, I will provide a complete dispersion analysis of these models, defining and characterizing the notion of negative (or negative index) materials [1]. In the case of dissipative media, I will provide a quantitative analysis of the energy decay for the solution of the Cauchy problem [2,3]. I will explain how the construction and analysis of Perfectly Matched Layers must be revisited in the case of negative index materials [4].

In a second part, I will treat transmission problems involving a dispersive medium and a non dispersive one. First, I will consider the junction of two half-spaces and will address the question of the limiting amplitude principle, i.e. the long time behaviour of the solution when the medium is excited with a time harmonic source of given frequency [5,6]. This will be the opportunity to describe mathematically unusual phenomena such as plasmonic surface waves or interface resonance. I will also present recent results (not published yet) on waves guided by a slab of dispersive material embedded in a non dispersive one.

A common tool for the analysis of the above mentioned questions is spectral theory. Some of the results will be illustrated with the help of time domain numerical simulations.

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Plenary Talk **Tuesday, 10:00–10:45, Tulla LH**

The small data global well-posedness conjecture for defocusing dispersive flows

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Providing a better description of the global dynamics for nonlinear dispersive PDE's has been the focus of many experts in the field for a long time. One fundamental property in this regard is *scattering*, which asserts that asymptotically the solutions to the nonlinear flow approach solutions to a corresponding linear flow. Scattering results have been proved for many nonlinear dispersive flows in cases where the nonlinear interactions are relatively mild due to the dispersive decay. However, there are also many models where the nonlinear interactions are too strong to allow for classical scattering, and where nonperturbative interactions are seen at all large time scales. Notably, this includes all cubic problems in 1D, for which global results have only been proved under the assumption that the initial data is both smooth, small and localized. However, except for the completely integrable case, until very recently no such results have been known for small but non-localized initial data, and indeed there was not even an indication that this might be at all possible.

This talk will present our recent conjecture, which broadly asserts that small data should yield global solutions for 1D defocusing dispersive flows with cubic nonlinearities, in both semilinear and quasilinear settings. So far we were able to prove the conjecture in several settings, which will be described. Finally, we will also discuss the higher dimensional counterpart of this conjecture, which is most interesting in 2D.

Plenary Talk **Wednesday, 9:00–9:45, Tulla LH**

Breathers and the Implicit Function Theorem

C. Eugene Wayne[1,*]
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Breather solutions are spatially localized, temporally periodic solutions of partial differential equations and lattice systems. Due to resonances between the breather frequency and the dispersive modes of the equation which cause a decay of localized oscillations, they are extremely rare for partial differential equations with constant coefficients. However, they are known to exist in some PDEs with inhomogeneous coefficients in which the inhomogeneity produces gaps in the spectrum that prevent these resonances from occurring.

In this talk I'll describe a new approach to the construction of breathers for special classes of PDEs with spatially periodic coefficients, based on the implicit function theorem. When studying periodic solutions of nonlinear wave equations on bounded intervals with the KAM theorem it proved very useful to first make assumptions about the properties of the spectrum of the linearized equation which insured the existence of periodic solutions, and then verify these hypotheses *a-posteriori*. In a similar vein I will define conditions for a nonlinear wave equation on a line which guarantee that the equation possesses breathers. As far as possible, these will be phrased in terms of the spectral properties of the linearized equation. One can then try to check that these properties are satisfied for various classes of equations.

Plenary Talk **Wednesday, 10:00–10:45, Tulla LH**

Cloaking fields and twinning spectra

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The proposal and subsequent realisation of a first invisibility cloak for microwaves in 2006 [1] opened the way towards structured materials which behave like curved spaces. So-called transformational optics has been extended to other types of waves (acoustic, hydrodynamic, seismic), as well as to diffusion processes (notably thermal [2]). The equations of the Scottish physicist James-Clerk Maxwell – which prefigured Albert Einstein's equations of relativity at the end of the 19th century – describe the propagation of an electromagnetic wave in a material are equivalent, under certain conditions, to those describing a wave on the surface of a liquid or a seismic surface wave. The techniques underlying invisibility (coordinate changes in mathematical equations that lead to anisotropic parameters that are realized in practice with structured media) build bridges between different disciplines of physics ranging from telecommunications to civil engineering.

It turns out that these techniques also apply to the creation of isospectral closed cavities: two cavities of different shapes and volumes can share the same natural frequencies, provided that at least one of them is made up of an anisotropic medium [3]. We can then dream of a recording studio with the acoustics of a cathedral.

Open cavities represent a further challenge as their spectrum is not purely discrete and real. Nonetheless, the twinning methodology developed for closed cavities is numerically shown to work in unbounded domains [4], making use of perfectly matched layers, and we pay special attention to the accurate twinning of leaky modes associated with complex-valued eigenfrequencies with an imaginary part orders of magnitude lower than the real part.

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Plenary Talk **Thursday, 9:00–9:45, Tulla LH**

The Control of Scattered Field for Linear and Nonlinear Scattering Media

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Many imaging methods in inverse scattering rely on the ability to superimpose scattering data so that the resulting scattered field corresponds to that of a point source. In the frequency domain, for compact inhomogeneities, this leads to solving two elliptic PDEs in a bounded region with a prescribed difference in Cauchy data. The case of zero scattered field leads to the study of the transmission eigenvalue problem and the regularity of free boundaries.

In this talk, we introduce this concept for linear media and review key results on the transmission eigenvalue problem, its resolvent, and non-scattering phenomena [1]. We then present recent results from [2] on the scattering problem for a nonlinear medium with compact support in the second-harmonic generation. When such a medium is probed with monochromatic light beams at a frequency ω, it generates additional waves at the frequency 2ω. The response of the medium is governed by a system of two coupled semilinear PDEs. We explore the possibility that the generated 2ω wave remains localized within the support of the medium, effectively rendering the nonlinear interaction with the probing wave invisible to an outside observer.

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Plenary Talk	**Thursday, 10:00–10:45, Tulla LH**

Successes and challenges of numerical time-domain boundary integral equations

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It is now respectively around four [1, 2] and three decades [3] since the publication of three fundamental papers giving a rigorous mathematical basis for the analysis of two particular classes of numerical methods for time-domain boundary integral equations: space-time Galerkin and convolution quadrature methods (CQ). Subsequently, there has been much further development of the theory applied to different problems building on these foundations. At the same time, there has been much work on fast and highly accurate numerical methods without a complete mathematical analysis, that nevertheless in numerical experiments have been shown to be effective, accurate and fast. This talk will give some new results that within the confines of convolution quadrature methods are good example of the fully theoretically grounded and a more pragmatic approach, but will also describe some modificitations that while effective are difficult to fully analyse.

We will revisit one of the fundamental papers published a decade ago [4] on FEM/BEM coupling for acoustics. The original approach required a certain stabilisation term, which was shown to be unnecessary in a second order formulation presented in [5]. We will describe a recent work [6] on a first order formulation addressing both the Maxwell and acoustic case and again not requiring the stabilisation term. We present this as an example of the power of the theoretical approach initially proposed in [3] and developed further in [4] and other works.

One of the weaknesses of the CQ approach for waves is its dissipative and dispersive character. While this can be dramatically improved by the use of high order Runge-Kutta methods, it is of interest to consider a simple modification of low-order methods that can produce much more accurate results. While this particular modification destroys the ability to fully analyse the method by standard means, the numerical results show both high accuracy and numerical stability [7].

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Plenary Talk **Friday, 9:00–9:45, Tulla LH**

The distorted Fourier transform, and the linearized Ginzburg-Landau operator

Jonas Lührmann[1], **Wilhelm Schlag**[2,*], **Sohrab Shahshahani**[3]

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This talk will begin with a review of the classical Weyl-Titchmarsh spectral theory. We will sketch how the distorted Fourier transform of a Schrödinger operator and the associated Plancherel theorem are obtained from the Weyl m-function via Stone's formula. Much less is known for non-selfadjoint differential operators. These can arise in the asymptotic stability theory of solitons and topological solitons [1,3,5]. A particular instance are vortex solutions of the complex Ginzburg-Landau equation in the plane from super-conductivity.

Recently, Gravejat, Pacherie, and Smets [2] established the orbital stability of the degree 1 vortex under the flow of the associated nonlinear Schrödinger equation, known as the Gross–Pitaevskii equation. For the more delicate asymptotic stability question, the first step is to investigate the linear Schrödinger evolution generated by the (non-selfadjoint) linearized matrix operator around the 1-vortex.

In the main part of the talk we will discuss ongoing work with Lührmann and Shahshahani [4] on the spectral and scattering theory of this class of matrix operators on the half-line. A delicate feature of the linear theory arises via the gauge invariance of the Ginzburg-Landau equation. This symmetry leads to an embedded resonance of the linearized operator with remarkable consequences for the dynamics. In fact, the resonance at zero energy forces the L^2 operator norm of the evolution to grow linearly in time.

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Plenary Talk **Friday, 10:00–10:45, Tulla LH**

Quantitative passive imaging of the interior of the Sun

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Helioseismology is the study of the interior of the Sun given observations of oscillations of the solar surface. Much research focuses on the reconstruction of flows in the convection zone (the outer 30% of the Sun), but also density, pressure, and other quantities are of interest. Observations consist of images of the line-of-sight velocities of the solar surface obtained via Doppler shift measurements, called dopplergrams. High resolution dopplergrams have been recorded continuously at a cadence of 45 seconds by satellite and ground-based instruments since more than 25 years.

Mathematically, solar oscillations are described by wave equations in the interior of the Sun, which are usually formulated in the frequency domain. Excitation, mainly caused by turbulent convection, is modelled as a random process. We consider the inverse problem to reconstruct coefficients of this wave equation given by correlations – or the covariance operator in the idealized noise free case – of the (random) solution to the wave equation restricted to the visible part of the surface of the Sun.

We report on uniqueness results stating that under certain reasonable assumptions covariance data uniquely determine the unknown coefficients of interest of the wave equation.

Numerical reconstructions are challenging since correlation data in helioseismology are extremely noisy and high dimensional. They depend of 5 variables (one frequency variable and 2x2=4 surface variables), and it is computationally unfeasibly to fully compute and store them in a preprocessing step. Classically, quantitative reconstructions in local helioseismology are achieved by computing a moderate number of linear functionals of the correlation data which can, e.g., be interpreted as travel times of certain types of waves, and then solving inverse problems with such reduced data. However, it can be shown that these procedures waste lots of the information contained in the correlation data. As an alternative, helioseismic holography has been used successfully since more than two decades to compute qualitative images of, e.g. the far side of the Sun, without the need of computing correlation data as preprocessing step. We present a new mathematical interpretation of (certain versions of) helioseismic holography, and based on this interpretation we propose an iterative method which is provably convergent. In particular, it provides not only qualitative, but also quantitatively correct reconstructions of the solar interior and the far side. It takes advantage of the full information content of correlation data without the need to compute these data in a preprocessing step, and as a consequence, it achieves significantly improved resolution compared to additional approaches.

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Plenary Talk **Friday, 11:30–12:15, Tulla LH**

Boundary Integral Exterior Calculus

Ralf Hiptmair[1,*], **Stefan Kurz**[1], **Erick Schulz**[2]
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We report a surprising and deep structural property of boundary integral operators occurring in first-kind boundary integral equations associated with Hodge–Dirac and Hodge-Helmholtz operators for de Rham Hilbert complexes on a bounded domain Ω in a Riemannian manifold. We show that, as regards their induced bilinear forms, those boundary integral operators are Hodge–Dirac and Hodge–Laplace operators in the weak sense, this time set in a trace de Rham Hilbert complex on the boundary $\partial\Omega$ whose underlying spaces of differential forms are equipped with non-local inner products defined through layer potentials. On the way to this main result we conduct a thorough analysis of layer potentials in operator-induced trace spaces and derive representation formulas.

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Minisymposia

List of Minisymposia

Minisymposium 01

Dissipative Patterns and Waves

Organizers: Joannis Alexopoulos and Björn de Rijk

MS01 **Wednesday, 11:30–11:55, SR 2.066**

On asymptotically self-similar patterns in coupled systems

Stefanie Schindler[1,*], **Alexander Mielke**[1]

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Self-similar behavior is a well-studied phenomenon in extended systems. However, the consideration is often restricted to scalar problems having exact self-similar solutions (e.g. the porous medium equation) or to problems with a trivial behavior at infinity. In this talk, we study a reaction-diffusion system and other related dissipative systems on the whole real line with prescribed non-trivial limits at infinity to investigate their long-term behavior. The system under consideration has the special property that it possesses a continuum of constant solutions and thus behaves asymptotically self-similar. In particular, no traveling waves occur. The key idea is to rescale space and time into parabolic scaling variables $\tau = \log(t{+}1)$ and $y = x/\sqrt{t{+}1}$ and to derive energy-dissipation estimates for a relative entropy to show that the solutions converge exponentially for $\tau \to \infty$ to self-similar profiles. In the original variables, these profiles correspond to asymptotically self-similar behavior describing the phenomenon of diffusive mixing of the different states at infinity.

MS01 **Wednesday, 12:00–12:25, SR 2.066**

Stability of discrete shock profiles for systems of conservation laws

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This talk deals with the stability analysis of discrete shock profiles for systems of conservation laws. These profiles correspond to approximations of shocks of systems of conservation laws by conservative finite difference schemes. Such discontinuous solutions appear naturally in the study of systems of conservation laws, which can model many physical situations, such as gas dynamics. Existence and stability of discrete shock profiles for each stable shock of the approximated system of conservation laws is seen as an improved consistency condition and implies that the finite difference scheme should approach discontinuities fairly precisely.

The aim of the talk will be to review some stability results regarding discrete shock profiles and to present a recent effort to extend them. More precisely, most results known up until recently are focused on the stability of discrete shock profiles associated with shocks of small amplitude. The talk will focus on a nonlinear stability result for discrete shock profiles in quite a general setting, where the smallness assumption on the shock's amplitude is replaced by a spectral stability assumption on the linear operator obtained by linearizing the numerical scheme about the discrete shock profile. This nonlinear stability result relies on a precise description of the Green's function of the linearization about discrete profiles presented in [1].

References

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MS01 **Wednesday, 12:30–12:55, SR 2.066**

Nonlinear stability of periodic waves trains against fully nonlocalized perturbations

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Many dissipative systems modeling important phenomena in biology, chemistry or physics exhibit a rich amount of patterns of which the most fundamental are periodic wave trains. Paradigm models of such dissipative pattern-forming systems are the FitzHugh-Nagumo system, reaction-diffusion systems, the Lugiato-Lefever equation and the St. Venant equations.

Proving nonlinear stability from spectral properties for such patterns is typically challenging since the spectrum touches the imaginary axis in 0. Additional challenges occur when we extend to fully nonlocalized perturbations, which do not obey any (co-)periodicity assumptions.

In my talk, I will present general approaches and tools that we have developed over the past years to tackle these issues along the aforementioned paradigm models.

MS01 **Wednesday, 15:00–15:25, SR 2.066**

Two-Time Scale Dynamics of Solutions to a Rimming-Flow Equation

Juri Joussen[1,*], **Christina Lienstromberg**[1], **Juan J. L. Velázquez**[2]
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We consider a thin layer of an incompressible viscous Newtonian fluid coating the inner wall of a horizontal cylinder rotating with constant speed. Assuming the film height is small compared to the radius of the cylinder, we formally derive a closed equation for the height $h(t,\theta) > 0$ of the liquid film by means of a lubrication approximation:

$$h_t + h_\theta + \gamma \left(h^3(h_{\theta\theta\theta} + h_\theta)\right)_\theta = g \left(h^3 \cos\theta\right)_\theta \qquad \text{in } (0,T) \times \mathbb{T}$$

This *rimming-flow* equation is of fourth-order, degenerate-parabolic, and quasilinear. Competing effects are observed between viscosity, the surface tension γ, and gravity g.

For $g = 0$ and a fixed mass m, positive travelling wave solutions with speed matching the cylinder rotation evolve only on the two-dimensional manifold

$$\mathcal{M}(m) := \left\{ m + a\sin\theta + b\cos\theta \;\middle|\; a^2 + b^2 < m^2 \right\}.$$

If $0 < g \sim \delta \ll 1$ is small, we show that solutions which are bounded away from zero converge exponentially fast to a δ-neighbourhood of $\mathcal{M}(m)$. Here, the existence of solutions on a large time scale $t \sim 1/\delta^2$ can be shown. Moreover, such solutions evolve on two distinct time scales: On the fast time scale t they only rotate around the origin with the speed of the cylinder, while on the slow time scale $\tau = \delta^2 t$ the dynamics are governed by an ODE in τ on $\mathcal{M}(m)$.

MS01 **Wednesday, 15:30–15:55, SR 2.066**

Sobolev Stability for the 2D MHD Equations in the Non-Resistive Limit

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The magnetohydrodynamic (MHD) equations model an incompressible and elec- trically conducting fluid. We are interested in the effects of mixing and quantitative stability of the MHD equations. Therefore, we consider perturbations of a combi- nation of an affine shear flow and a constant magnetic field in a periodic channel. For the partial dissipation regime, when magnetic resistivity is smaller than fluid viscosity $\nu \geq \kappa > 0$, our main result is the nonlinear stability of perturbations that are small enough in Sobolev spaces. Furthermore, if $\kappa \leq \nu^3$ the system exhibits norm inflation of the Sobolev norm by $\nu\kappa^{-\frac{1}{3}}$. Thus the norm inflation increases with the viscosity.

MS01 **Thursday, 15:00–15:25, SR 3.068**

Pattern formation in reaction-diffusion systems with spatially periodic coefficients

Jolien Kamphuis[1,*], **Martina Chirilus-Bruckner**[1]
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We are interested in the formation of patterns that arise as solutions to nonlinear partial differential equations with spatially varying coefficients. We focus on one-dimensional patterns that are described by the evolution of modulated periodic orbits that bifurcate from a background state in the presence of a spatially varying heterogeneity. These nonlinear PDEs with spatially varying coefficients emerge from application based models, such as ecology models, where the spatially varying coefficients represent a spatially varying terrain. In this case, the corresponding solutions can be interpreted as vegetation patches adjusting to the periodic domain, hence forming patterns on patterns.

During this talk, we'll showcase along various examples of PDEs with spatially periodic coefficients, such as Klausmeier and Swift-Hohenberg, how Bloch analysis can be used to derive the corresponding modulation equation. An additional challenge is caused by resonance phenomena, hence we have to distinguish between resonant and non-resonant cases, based on the wavenumber of the forcing.

MS01 **Thursday, 15:30–15:55, SR 3.068**

Detecting Dissipative Wave Phenomena in Half-Laplacian PDEs Using a Parallel Boundary Value Method

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In this presentation, we address the following time-dependent PDE [1]:

$$u_t = -\epsilon(-\Delta)^{\frac{1}{2}}u + \mathscr{L}(u) + f(x,t), \epsilon > 0,\ (x,t) \in (0,L) \times (0,T),$$

where $\mathscr{L}$ is a linear differential operator and f is a source term. Different from previous approaches, we base our method on the relationship between the Hilbert transform and the half-Laplacian to propose a doubling-splitting method, which results in a backward wave equation (BWE) where the half-Laplacian term is considered only in the initial condition. We then apply a second-order parallel boundary value method [2] for this time-dependent problem over a large time scale, showing that the method is convergent and stable even in ill-posed cases.

This work aims to explain two cases: an advection-dominated equation and the space-fractional Schrödinger equation [3]. We show that the solution to the BWE is equivalent to that of the original PDE, both analytically and numerically. This result provides the special dissipation, half-diffusion, is embedded in traveling solutions and can be explained through the BWE.

References

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Minisymposium 02

Numerical Methods for Nonlinear and Nonlocal Transient Phenomena

Organizers: Organizers: Lehel Banjai and María López-Fernández

MS02	**Tuesday, 11:30–11:55, SR 0.014**

Runge-Kutta convolution quadrature based on Gauss methods

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In this talk we consider a discretization technique for boundary integral operators in the time domain: Convolution Quadrature (CQ) combined with Galerkin BEM or spatial collocation. The discussion focuses on recent results on CQ schemes based on Gauss Runge-Kutta methods. We present an error analysis for these methods applied to hyperbolic operators. Order reduction is observed, with the order of convergence depending heavily on the parity of the number of stages, a more favourable situation arising for the odd cases than the even ones. An exception is observed when the associated kernel exhibits exponential decay. In this case, for the 2-stage Gauss method full order is obtained. For particular situations the order of convergence is higher than for Radau IIA or Lobatto IIIC methods when using the same number of odd stages. We investigate an application to transient acoustic scattering where, for certain scattering obstacles, the favourable situation occurs in the important case of the exterior Dirichlet-to-Neumann map. Numerical experiments and comparisons illustrate the performance of the method.

References

[1] L. Banjai and M. Ferrari, Runge-Kutta convolution quadrature based on Gauss methods, *Numer. Math.*, published online, (2024).

MS02 **Tuesday, 12:00–12:25, SR 0.014**

Eliminating artificial boundary conditions for the time-dependent Schrödinger equation using Fourier contour deformation

Jason Kaye[1,*], **Alex Barnett**[2], **Leslie Greengard**[2], **Umberto de Giovannini**[3], **Angel Rubio**[4]

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I will introduce a method for solving the time-dependent Schrödinger equation in free space with a compactly-supported time-dependent potential. It avoids artificial boundary conditions using a pseudospectral-type method with the real Fourier axis of each spatial dimension deformed to a complex contour, yielding the free space solution restricted to a computational subdomain with controlled accuracy. The scheme is high-order accurate in space and time, has quasi-optimal computational complexity, operates on Cartesian spatial grids in any dimension, permits spatially-uniform applied fields, and allows solutions to leave the computational domain and return later. I will present results from simulations of absorption and photoemission spectroscopy using time-dependent density functional theory which suggest the method enables the use of significantly smaller computational domain sizes than the complex absorbing potentials presently in common use.

MS02 **Tuesday, 12:30–12:55, SR 0.014**

A posteriori error control for an energy preserving method for the Schrödinger-Poisson system

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In this talk we first introduce a structure preserving, second order in time relaxation-type scheme for approximating solutions of the Schrödinger-Poisson system and highlight its advantages. We then discuss the a posteriori error analysis for this scheme. In particular, we introduce an appropriate reconstruction and present the main steps of the derivation of optimal order a posteriori error bounds. The main challenges in the analysis arise from the nonlinear nature of the problem and the fact that the potential function, which is also unknown, is linked with the wave function. This means that we should obtain optimal a posteriori error estimates for both the potential and the wave function at the same time. Various numerical experiments verify and complement our findings.

MS02 **Tuesday, 15:00–15:25, SR 1.067**

Westervelt-based modeling of ultrasound-enhanced drug delivery

Julio Careaga[1], Vanja Nikolić[2,*], Belkacem Said-Houari[3]
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[2] *Radboud University, The Netherlands*
[3] *University of Sharjah, United Arab Emirates*
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In this talk, we will discuss a nonlinear and nonlocal multiphysics model motivated by ultrasound-enhanced drug delivery. To adequately capture the nonlinear effects in ultrasound propagation through soft tissue, the acoustic pressure field is modeled by Westervelt's quasi-linear wave equation with damping of the time-fractional type. Additionally, acoustic medium parameters are allowed to depend on the temperature of the medium. This wave equation is then coupled to the nonlinear Pennes equation with a pressure-dependent source to account for ultrasound waves heating the tissue. Finally, the drug concentration is obtained as the solution of an advection-diffusion equation with a pressure-dependent velocity. Toward gaining a rigorous understanding of this system, we set up a fixed-point argument in the analysis combined with devising energy estimates that can accommodate the time-fractional attenuation. The energy arguments are, in part, carried out by employing time-weighted test functions to reduce the regularity assumptions on the initial temperature. In addition to these theoretical considerations, we will also illustrate the properties of this system through numerical experiments.

MS02 **Tuesday, 15:30–15:55, SR 1.067**

The p-version of convolution quadrature in wave propagation

Alexander Rieder[1,*]
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In this talk, we present a novel approach towards boundary element methods for wave propagation. It is based on the convolution quadrature idea by Lubich [2], but instead of relying on reducing the timestep size in order to achieve higher accuracy, we use the p-refinement paradigm of increasing the order of the method while keeping the timestep size fixed. To get an easily computable and analyzable scheme, we rely on the ideas of discontinuous Galerkin timestepping [3]. This allows us to design a scheme which is root-exponentially convergent for certain very smooth initial conditions. We talk about recent progress in analyzing this new scheme, as well its practical implementation and challenges.

References

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MS02 **Tuesday, 16:30–16:55, SR 1.067**

High-order Exponential integrators for semilinear subdiffusion equations with rough initial data

Buyang Li[1], **Yanping Lin**[1], **Shu Ma**[2,*], **Qiqi Rao**[1]
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An exponential type of convolution quadrature is proposed as a time-stepping method for the semilinear subdiffusion equation with bounded measurable initial data. The method combines contour integral representation of the solution, quadrature approximation of contour integrals, multistep exponential integrators for ordinary differential equations, and locally refined stepsizes to resolve the initial singularity. The proposed k-step exponential convolution quadrature can have kth-order convergence for bounded measurable solutions of the nonlinear subdiffusion equation based on natural regularity of the solution with bounded measurable initial data. Recently, a new spectral method is also constructed for the semilinear subdiffusion equations with possibly discontinuous rough initial data.

MS02 **Tuesday, 17:00–17:25, SR 1.067**

Numerical algorithms for nonlinear wave equations via De Giorgi's approach

Brian Hennessy[1,*], **Emmanuil Georgoulis**[1], **Lehel Banjai**[1]
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In this talk we present algorithms for numerically solving nonlinear wave (NLW) equations based on a variational regularisation approach conjectured by Ennio De Giorgi[1]. This approach guarantees existence and uniqueness to a solution of a regularised nonlinear wave equations which converge to a solution of NLW. This conjecture was later proved in the finite time case by Stefanelli using a time discretisation argument which naturally leads to numerical algorithms which will be discussed[2].

We also present methods which discretise the problem via spacetime finite elements with particular attention being paid to the time discretisation.

References

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[2] Ulisse Stefanelli. "The De Giorgi conjecture on elliptic regularization". In: Mathe- matical Models and Methods in Applied Sciences 21.06 (2011), pp. 1377–1394. doi: 10.1142/S0218202511005350.

MS02 Tuesday, 17:30–17:55, SR 1.067

Patterns and waves in the space-fractional Gray-Scott system

Paul Andries Zegeling[1,*], Pu Yuan[1]
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We describe an efficient new numerical method for solving space-fractional partial differential equations (PDEs) of fractional order $\alpha \in (1, 2)$ on non-uniform adaptive finite difference meshes. The fractional Laplacian in the PDE system is defined in terms of Riemann–Liouville fractional derivatives. Note that, for $\alpha = 2$, the original integer-order PDE is retained, whereas, for $\alpha = 1$, the Hilbert transform $\mathcal{H}$ comes into play. Our approach as described in detail in [2] extends the so-called L2 method to the non-uniform mesh case. The spatial mesh generation makes use of adaptive moving finite differences, offering adaptation at each time step through grid reallocation based on previously calculated solutions. The chosen adaptive mesh movement technique, MMPDE5, demonstrates rapid and efficient mesh movement, yielding accurate numerical solutions, especially for the cases with steep moving transitions. Numerical experiments for the space-fractional Gray-Scott reaction-diffusion model, reveal a rich set of different patterns, showing interesting and surprising differences in behaviour, compared to the integer-order case as discussed in [1]. The adaptive method detects self-replication patterns, travelling waves, and chaotic solutions, along with two remarkable evolution processes depending on the order α: from self-replication to standing waves and from travelling waves back to self-replication.

References

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Minisymposium 03

Intersection of Biomedical Acoustics and Machine Learning

Organizers: Marta Betcke , Ben Cox, and Matthew King

MS03	Monday, 16:30–16:55, SR 3.068

Self-supervised sparse-data photoacoustic image reconstruction

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In photoacoustic imaging, acquiring aa sparse dataset leads to standard reconstructions with severe artifacts and noise. To reduces these issues, we propose a self-supervised image reconstruction strategy, inspired by the approach in [1] for sparse-view CT. Our method leverages an self-supervised loss function defined in the projection domain, allowing to learn the nullspace component. We demonstrate that this approach effectively reduces undersampling artifacts and noise leading to improved reconstruction quality.

References

[1] N. Gruber, J. Schwab, E. Gizewski and M. Haltmeier, Sparse2Inverse: Self-supervised inversion of sparse-view CT data, arXiv:2402.16921 (2024).

MS03 **Monday, 17:00–17:25, SR 3.068**

Fast Fourier models in circular geometry for learned reconstructions in photoacoustic tomography

Janek Gröhl[1], **Andreas Hauptmann**[2,*], **Leonid Kunyansky**[3], **Jenni Poimala**[4]
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Learned reconstructions have been highly successful in recent years, nevertheless training and deployment of such data-driven approaches is computationally highly demanding. This is especially the case for so-called model-based learned iterative reconstructions, which intertwine neural network components with the forward and adjoint operator [1]. Consequently, the computational footprint is primarily governed by the model components, which necessitates efficient forward and adjoint models.
In this work we investigate the use of fast Fourier based methods in the circular geometry. The first such method was based on a fast implementation of an inverse operator [2]. We start by discussing this algorithm and its application to experimental data. Further, we present the fast forward and the corresponding adjoint formulation, which enables efficient implementation of learned iterative reconstructions.

References

[1] A. Hauptmann et al., Model-based learning for accel- erated, limited-view 3-d photoacoustic tomography, *IEEE transactions on medical imaging*, **37** (2018), pp. 1382–1393.
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MS03 **Monday, 17:30–17:55, SR 3.068**

A Hybrid Gaussian Beam Method for the PAT Inverse Problem

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Photoacoustic tomography (PAT) is a hybrid imaging technique based on the photoacoustic effect. The PAT forward problem can be modelled as an initial value problem for the free space wave equation. The PAT inverse problem aims to recover an initial pressure from pressure time series recorded at sensors placed outside the region of interest. We are able to efficiently solve the PAT forward problem using the Multiscale Gaussian Beam method proposed by Qian and Ying [1]. We developed a hybrid solver, which used `k-Wave` [2] to efficiently and accurately represent the low frequency components. Now, using the same Multiscale Wavepacket transform to efficiently decompose the pressure time series [3], we propose a hybrid solver to solve the PAT inverse problem using a time reversal approach.

References

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[2] B. E. Treeby and B. T. Cox, k-Wave: MATLAB toolbox for the simulation and reconstruction of photoacoustic wave fields, *Journal of Biomedical Optics* **15** (2010), pp. 021314–021314.
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MS03 Monday, 18:00–18:25, SR 3.068

Tackling the photoacoustic inverse problems with semi-supervised deep learning

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Photoacoustic imaging promises functional characterisation of biological tissues, but the associated optical and acoustic inverse problems are challenging and hinder the quantification of derived biomarkers. One reason for this is the approximate nature of the forward models, often referred to as the *simulation gap* between simulated and experimental data [1]. In this work, we present a two-stage semi-supervised deep learning approach to tackling the inverse photoacoustic problems.

We first use time series-domain data to close the *simulation gap* of the forward model by adding a post-processing neural network that is trained using a semi-supervised approach. In the second stage, we train a supervised network on a hybrid dataset with experimental and simulated images for the optical inverse problem, after approximating the acoustic inverse problem with a conventional reconstruction scheme.

Our results show that we can effectively close the simulation gap and transform the qualitative photoacoustic images into a quantitative domain highly correlated with the optical absorption coefficient distribution.

References

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MS03 Tuesday, 11:30–11:55, SR 3.068

Physics-informed deep learning for ultrasonic imaging

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Ultrasonic (US) imaging is being used in many clinical and industrial applications to obtain information about the acoustic properties of a medium by emitting waves into it and recording their interaction using ultrasonic transducer arrays. In the past decade, enormous improvements in US image quality have been achieved by introducing deep-learning-based methods. Predominantly, image-to-image post-processing networks or fully learned data-to-image neural networks are used. Both construct their mapping purely data-driven and require expressive networks and large amounts of training data to perform well.

In contrast, we established a line of work that incorporates conventional, wave-physics-based image formation algorithms such as delay-and-sum or f-k migration as differentiable layers in the network architecture to create light-weight, data-driven approaches that can be trained end-to-end towards a given imaging task with small amounts of training data.

We demonstrate the effectiveness of our approach on simulated and experimental data and for tasks such as data compression, multi-view fusion and segmentation and close with an in-depth evaluation of ultrafast ultrasound imaging using single plane waves with an experimental data collection from realistic breast mimicking phantoms and ultrasound calibration phantoms. The evaluation considers global and local image similarity measures and contrast, resolution and lesion detectability analysis.

MS03 Tuesday, 12:00–12:25, SR 3.068

Techniques to Overcome Cycle Skipping in Frequency-Domain Full-Waveform Inversion for Ultrasound Tomography

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Ultrasound tomography fundamentally depends on low-frequency data to avoid cycle skipping in full-waveform inversion. In the absence of sufficiently low-frequency data, we can extrapolate the necessary low-frequency content using a variety of approaches [1][2]. This low-frequency content can then be used to kickstart full-waveform inversion without cycle skipping. We demonstrate the efficacy of these low-frequency extrapolation techniques in a series of experiments that progress from *in-silico* simulations and *in-vitro* phantoms to *ex-vivo* brain samples and *in-vivo* breast imaging cases. In each experiment, internal structures are visible with much greater clarity without cycle skipping artifacts when low-frequency extrapolation is used to kickstart full-waveform inversion.

References

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MS03 Tuesday, 12:30–12:55, SR 3.068

Neural-FMM: Integrating Traditional Solvers with Learnable Operators

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We propose a novel neural network architecture, Neural-FMM, that integrates the Fast Multipole Method [1] (FMM) into a hierarchical machine learning framework for learning the Green's operator of elliptic PDEs. Our architecture leverages the FMM's hierarchical computation flow to efficiently separate local and far-field interactions, learning their respective representations. The Neural-FMM replaces the traditional hand-crafted FMM translation operators with deep feedforward neural networks while maintaining non-local information flow. We will also discuss modifications to handle non-stationary kernels and numerical experiments on inhomogeneous and heterogeneous Helmholtz equations to demonstrate the Neural-FMM's effectiveness in solving scattering problems with variable sound speed maps and incident fields.

References

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Minisymposium 04

Asymptotic Models in Fluid Dynamics and Pattern Formation

Organizers: Gabriele Brüll, Christina Lienstromberg, and Wolf-Patrick Düll

MS04	Wednesday, 16:30–16:55, SR 1.067

2D Stokes Problem With a Moving Contact Line

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We consider the evolution of a two–dimensional liquid droplet on a solid subtrate in the presence of a contact point. At the liquid-solid interface we assume Navier slip. At the triple point we assume a constant contact angle, given by Young's law. The main result is well–posedness and regularity of the solution in a class of weighted Sobolev spaces.

MS04 **Wednesday, 17:00–17:25, SR 1.067**

Dynamics of Solutions to a 2D Rimming-Flow Equation

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A *rimming flow* is a fluid flow in a partially filled, horizontal cylinder which rotates around its axis. We consider an incompressible Newtonian fluid coating the inner wall of the cylinder and model the full 3D problem by the incompressible Navier–Stokes equations. In the case of a very thin fluid film with surface tension being very large compared to gravity, we formally derive the following 2D rimming flow equation for the film height $h(t, \theta, z)$, $t > 0$, $\theta \in \mathbb{T}$, $z \in (0, L)$:

$$h_t + h_\theta + \gamma \operatorname{div}\left(h^3 \nabla(\Delta h + h)\right) = g\left(h^3 \cos(\theta)\right)_\theta \qquad \text{in } (0,T) \times \mathbb{T} \times (0,L).$$

This is a quasilinear and fourth-order degenerate-parabolic equation.

We discuss significantly different phenomena compared to the 1D case, e.g. non-uniqueness of steady states whenever the length L of the cylinder satisfies $L \in \pi\mathbb{N}$.

This talk is based on ongoing joint work with Janne Laudien (Stuttgart), Christina Lienstromberg (Stuttgart), and Juan J. L. Velázquez (Bonn).

MS04 **Wednesday, 17:30–17:55, SR 1.067**

Optimal control of thin film droplet dynamics

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Interfacial instabilities in volatile liquid thin films on a hydrophobic substrate can lead to complex droplet dynamics such as droplet merging, splitting, and transport. We study a class of mean field control formulations for these droplet dynamics. An optimal control problem is designed by formulating the droplet dynamics as gradient flows of free energies in modified optimal transport metrics with nonlinear mobilities. As an example, we consider a thin volatile liquid film laden with an active suspension, with control achieved through its activity field. Numerical examples, including droplet transport, bead-up/spreading, and merging/splitting on a two-dimensional spatial domain, demonstrate the effectiveness of the proposed mean field control mechanism. This is joint work with Guosheng Fu, Will Pazner, and Wuchen Li.

MS04 Wednesday, 18:00–18:25, SR 1.067

Solutions of the thin film equation with shrinking support

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In this talk I will discuss several solutions of the thin film equation, with the usual mobility h^3, for which the support of the solutions shrinks in finite time. The meaning of the dissipation of energy formula will be discussed in detail. Some particular self-similar solutions describing mass aggregation will be described too.

MS04 Thursday, 11:30–11:55, SR 2.066

A variational view on constitutive laws in time-dependent problems

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We present a variational approach to the study of non-Newtonian fluids. Where classical modelling relates the viscous stresses σ in the fluid to the shear rates ϵ by means of a fixed constitutive law, $\sigma = \sigma(\epsilon)$, we introduce a distance measure $f(\epsilon, \sigma)$ that encodes how much a given strain-stress pair deviates from an underlying relationship. If we have measurement data $\mathcal{D} = \{(\epsilon_j, \sigma_j)\}$ available, we can define f as (a power of) the distance to $\mathcal{D}$. This way, we avoid any ad hoc construction of a constitutive law and follow a data-driven paradigm. From this perspective, the study of the non-Newtonian Navier–Stokes system becomes a problem of minimising the functional

$$(\epsilon, \sigma) \mapsto \iint_{(0,T)\times\Omega} f(\epsilon(t,x), \sigma(t,x))\,\mathrm{d}x\,\mathrm{d}t$$

subject to side-constrains of mass and momentum conservation of the corresponding fluid flow.

We will discuss necessary and sufficient growth and convexity conditions on f to ensure that minimisers exist. As we will see, these requirements translate into geometrical properties of the underlying data set and correspond to classical existence results from the PDE perspective.

MS04 **Thursday, 12:00–12:25, SR 2.066**

Pattern Formation in a 2D Thermocapillary Thin-Film Equation

Stefano Böhmer[1,*], **Bastian Hilder**[2], **Jonas Jansen**
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It is experimentally known that thin films of viscous fluids on heated plates develop polygonal, spatially periodic patterns. This is due to a self-sustaining thermocapillary effect causing an instability of the trivial constant state.

Building upon the 1D results in [1], we consider a two-dimensional thin-film equation. It can be retrieved as an asymptotic limit of the Boussinesq–Navier–Stokes model for small fluid height. The equation is of fourth order, quasilinear, and degenerate parabolic but we consider the stationary problem, which we are able to reduce to a second-order equation amenable to analytic bifurcation theory.

The constant solution destabilizes via a (conserved) long-wave instability and we prove existence of a global bifurcation branch of stationary solutions of fixed mass, which are symmetric and periodic with respect to a fixed square or hexagonal lattice. We finally analyse qualitative aspects of the solutions on the branch.

References

[1] G. Bruell, B. Hilder and J. Jansen, Thermocapillary thin films: periodic steady states and film rupture, *Nonlinearity* **37** (2024), 045016.

MS04 **Thursday, 12:30–12:55, SR 2.066**

Patterns and fronts in thermocapillary thin films

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Instabilities in thin fluid films heated from below have garnered significant attention since Bénard observed the formation of hexagonal patterns in the early 1900s. These instabilities are driven by thermocapillary effects, where temperature-dependent surface tension induces flow along the free surface. Subsequent experimental studies revealed the competition between two distinct instabilities in these systems: a long-wave instability, leading to film rupture and dewetting, and a short-wave instability, resulting in pattern formation in the wake of an invading front.

In this talk, I will present recent findings on spatially periodic patterns with hexagonal symmetry and the dynamics of moving pattern interfaces within an asymptotic thermocapillary thin-film model near a monotonic short-wave, or Turing, instability.

MS04 **Thursday, 16:30–16:55, SR 0.014**

Justification of the Nonlinear Schrödinger and the Davey-Stewartson approximation for the Three-Dimensional Water Wave Problem With and Without Surface Tension

Franz Schewe[1,*], **Wolf-Patrick Düll**[1]
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We consider the three-dimensional water wave equations in an infinite area with finite or infinite water depth and with or without surface tension. This system has wave-packet like solutions whose envelopes can formally be described by solutions to the Nonlinear Schrödinger equation in the case of infinite depth, or solutions to the Davey-Stewardson system in case of finite depth. While it was already shown that analogous approximations can be rigorously justified for the two-dimensional water wave problem, in three dimensions exist only approximation results for infinite depth and no surface tension.

In this talk, we discuss the rigorous approximation for the three-dimensional water wave problem in isothermal coordinates with and without surface tension over a physically relevant time span. We show uniform estimates for small values of the surface tension, and handle additional resonances that arise in three dimensions in order to get estimates on the required time scale. For the error estimates, we use a new systematic approach that uses the Hamiltonian structure of the original system.

MS04 **Thursday, 17:00–17:25, SR 0.014**

On a new construction of solitary waves

Katerina Nik[1,*], **Mats Ehrnstrom**[2], **Christoph Walker**[3]
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Starting with the periodic waves earlier constructed for the gravity Whitham equation

$$u_t + (Lu + u^2)_x = 0, \qquad \mathcal{F}(Lu)(\xi) = \left(\frac{\tanh(\xi)}{\xi}\right)^{\frac{1}{2}} \mathcal{F}(u)(\xi),$$

we parametrize the solution curves through relative wave height, and use a limiting argument to obtain a full family of solitary waves. The resulting branch starts from the zero solution, traverses unique points in the wave speed-wave height space, and reaches a singular highest wave. The construction is based on uniform estimates on periodic waves together with limiting arguments and a Galilean transform to exclude vanishing waves and waves leveling off at negative surface depth.

MS04 **Thursday, 17:30–17:55, SR 0.014**

Mathematical Aspects of the Stratified Ocean Flows

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[3]*Delft Institute of Applied Mathematics, TU Delft, Delft, The Netherlands*
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I will present a mathematical perspective concerning geophysical water flows exhibiting stratification, internal waves, and a preferred (azimuthal) propagation direction on an example of the Antarctic Circumpolar Current (ACC) [1, 2]. ACC is one of the major ocean currents on the Earth. It encircles the Southern ice-covered continent, being with an overall length of about 24000 km the longest oceanic current. It flows clockwise from west to east around Antarctica between latitudes 45°S and 55°S, where there are no land masses to interfere with this continuous stretch of water. Relatively slow, the ACC extends from the sea surface to depths of 2000-4000 m reaching, unlike other major currents, from the surface to the bottom of the ocean. ACC isolates Antarctica with a ring of cold water and is to a large extent responsible for Antarctic permanent glaciation.

References

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Minisymposium 05

Nonlinear Waves in Cardiac Tissue

Organizers: Martina Chirilus-Bruckner and Tim de Coster

MS05	**Tuesday, 16:30–16:55, SR 3.068**

Using multi-pulse protocols for the control of spiral wave chaos

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The chaotic spatio-temporal dynamics during cardiac arrhythmia is governed by a (chaotic) spiral or scroll wave dynamics. An energy efficient control of the wave dynamics is highly relevant in the medical context of defibrillation and cardioversion. We use numerical simulations as an ideal tool to test new control strategies in a safe environment. In contrast to the application of single high-energy pulses, multipulse protocols provide a promising approach to reduce the side effects of current treatments. We discuss and compare different approaches, including predetermined pulse sequences as well as feedback solutions.

References

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MS05 **Tuesday, 17:00–17:25, SR 3.068**

Spiral waves — some recent progress

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I will summarize somewhat selectively recent theoretical efforts at grappling with the notoriously intractable questions of existence, stability, and instability of spiral waves. I'll first give a brief summary of the conceptual viewpoint from work with Björn Sandstede, which views spiral waves as heteroclinic orbits asymptotic to a periodic orbit in an infinite-dimensional, ill-posed radial dynamical system. This view point allows one, to describe, with numerically verifiable assumptions, many intricate properties of the linearization at a spiral wave, the effects of boundaries, and predict qualitative features of potential instabilities. I'll then discuss more recent efforts towards a more complete theoretical understanding in a setting of anchored spirals, where the spiral arm attaches to a central disk-shaped hole in the domain. Models we analyze are both a sharp interface approximation and simple phase oscillator reaction-diffusion dynamics.

MS05 **Tuesday, 17:30–17:55, SR 3.068**

Accelerated simulation of cardiac tissue using data-driven models

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Faster than real time predictions of cardiac excitation patterns can open up new ways to warn about impending formation of arrhythmias and their prevention. Recently, we have introduced a novel method to create data-driven models for cardiac electrophysiology from 2D spatio-temporal recordings such as optical voltage mapping data. The models obtained from this fully automatic model creation pipeline are encoded as simple polynomials enabling computational simulations of cardiac tissue at high speed.

Directly utilising hardware capabilities and using optimised code, a cardiac emulator using our data-driven low-order predictive models can predict excitation waves in cardiac monolayers faster than real time. We propose a decision making algorithm managing the cardiac emulator in four phases: fitting the model, predicting, warning and preventing cardiac arrhythmias. Such a non-static model can be used to learn properties specific to each monolayer on the fly to then yield more relevant predictions, even if such properties change over the course of the lifetime of a tissue sample.

Extensions of this cardiac emulator to full heart geometry could be the next step towards a true personalised digital twin of the heart, holding immense potential to diagnose and treat cardiac rhythm disorders, bridging the gap between cardiac simulation of the heart and clinical practice.

MS05 **Tuesday, 18:00–18:25, SR 3.068**

Roles of Stretch-Activated Channels in Atrial Fibrillation: From Cellular Dynamics to Whole-Heart Simulations

Stephanie Appel[1,*], **Tobias Gerach**[1], **Jonathan Krauß**[1], **Axel Loewe**[1]

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Stretch-activated channels (SACs) may be involved in the pathogenesis of atrial fibrillation (AF), but underlying mechanisms are incompletely understood. This *in-silico* study assessed the impact of SACs at both the cellular and whole-heart levels. SACs were modeled according to Gerach and Loewe [1], and integrated into the atrial cell model of Courtemanche et al. [2]. A fully electro-mechanically coupled whole-heart framework [3] was used for tissue simulations. Uncertainty quantification was performed by varying SAC channel density and adjusting the tissue conductivity to account for the characteristics of AF-prone tissue. We found pronounced afterdepolarization in the upper atria an close to the atrioventricular valves related to high cellular stretch. Afterdepolarizations promoted atrial refractoriness and led to ectopic activity, potentially leading to reentrant activation and contributing to AF pathogenesis.

References

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Minisymposium 06

Mathematics of Acoustic Nonlinearity Tomography

Organizers: Ben Cox, Barbara Kaltenbacher, Felix Lucka, and Vanja Nikolić

MS06	**Thursday, 15:00–15:25, SR 2.066**

Well-posedness of a first order in time nonlinear wave equation with nonhomogeneous Dirichlet or Neumann type boundary conditions in fractional Sobolev spaces

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In this work, we analyze the well-posedness of a first order in time nonlinear wave equation with nonhomogeneous Dirichlet or Neumann type boundary conditions, also known as known as Hodge, Lions or Navier-slip boundary conditions, in fractional Sobolev spaces. We do this by first showing the well-posedness of an abstract system and then apply the results to concrete operators and function spaces that represent the different boundary conditions in an appropriate sense.

The analysis of the abstract system is based on a spectral decomposition of a symmetric operator. The regularity of the solution is given in terms of the domains of fractional powers of this operator. Using Galerkin's method and Newton-Kantorovich Theorem, we prove well-posedness of the abstract nonlinear system with possibly nonhomogeneous boundary conditions.

The connection between the domains of fractional powers of the operator and fractional Sobolev spaces makes it possible to obtain results for a system with nonhomogeneous Dirichlet boundary conditions with fractional Sobolev space regularity. For the Neumann type boundary conditions we show an integer valued Sobolev regularity of the solution.

MS06 **Thursday, 15:30–15:55, SR 2.066**

Nonlinear ultrasound contrast imaging - the Westervelt–Rayleigh–Plesset model

Teresa Rauscher[1,*], **Vanja Nikolić**[2]

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Ultrasound contrast imaging is a specialized imaging technique that applies microbubble contrast agents to traditional medical sonography providing real-time visualization of blood flow and vessels. Gas filled microbubbles are injected into the body where they undergo compression and rarefaction and interact nonlinearly with the ultrasound waves. Therefore, the propagation of sound through bubbly liquid is a strongly nonlinear problem that can be modeled by a nonlinear acoustic wave equation for the propagation of the pressure waves coupled with an ordinary differential equation for the bubble dynamics. We start by deriving different models and then focus on the coupling of the Westervelt equation and the Rayleigh-Plesset equation, where we show well-posedness locally in time under suitable conditions on the initial data. Finally, we present numerical experiments on the single bubble dynamics and the interaction of the microbubbles and ultrasound waves.

References

[1] T. Rauscher and V. Nikolić, The Westervelt-Rayleigh-Plesset model of ultrasound contrast imaging with microbubbles: analysis and simulation, https://arxiv.org/abs/2408.06108.

MS06 **Thursday, 16:30–16:55, SR 2.066**

Asymptotic-preserving HDG method for the Westervelt quasilinear wave equation

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In the recent work [1], we study the asymptotic-preserving properties of a hybridizable discontinuous Galerkin (HDG) method for the Westervelt model of ultrasound waves. More precisely, we show that the proposed HDG method is robust with respect to the sound diffusivity (damping) parameter δ, by deriving energy stability estimates and *a priori* error bounds that are independent of δ. Such bounds are then used to show that, when $\delta \to 0^+$, the method remains stable and the discrete acoustic velocity potential solution $\psi_h^{(\delta)}$ converges to $\psi_h^{(\delta=0)}$, which is the singular vanishing dissipation limit. Moreover, the method provides an optimally convergent approximation of the acoustic particle velocity variable $\boldsymbol{v} = \nabla\psi$.

This work is inspired by the analysis in [2, 3] for standard and mixed FEM.

References

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MS06 Thursday, 17:00–17:25, SR 2.066

Stability analysis of a new curl-based full field reconstruction method in 2D magnetic resonance elastography

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In Magnetic Resonance Elastography, the shear modulus of tissues μ is determined from full-field displacement data by solving an inverse problem based on the time-harmonic, linear, isotropic and nearly incompressible elastodynamic wave equation. The main challenge lies in developing a robust algorithm for handling noisy data. Specifically, the nearly incompressible nature of tissues means that the bulk λ is very high, which often causes straightforward extensions of existing algorithms to fail in maintaining robustness.

Following [1, 2], we introduce a new formulation for the 2D problem. We reformulate the problem as a non-autonomous hyperbolic system to establish existence, uniqueness, and stability estimates for the inverse problem. To mitigate the impact of noise, we have implemented a regularization technique [3]. Convergence properties of the method is evaluated using in-silico data.

References

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[2] Ammari H. et. al., 2015. Stability analysis for Magnetic Resonance Elastography.

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Minisymposium 07

Nonlinear Maxwell Equations

Organizers: Tomáš Dohnal and Wolfgang Reichel

MS07 **Wednesday, 16:30–16:55, SR 2.067**

Justification of amplitude equations for nonlinear Maxwell systems with memory.

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The framework of evolutionary equations provides a unified well-posedness, regularity, and stability theory for linear, time-invariant systems with or without memory. This theory was successfully applied and extended in [1] to dispersive Maxwell systems with nonlinearities of local and non-local type: On bounded domains and for a wide class of optical (non-magnetic) materials, exponential stability for the linearized system, as well as small, exponentially decaying solutions of the nonlinear system, were obtained. Based on these results, and on ideas in [2], where slowly modulated surface plasmonic modes on an unbounded domain were studied for suitable magneto-optical materials, in the present work [3], we consider primarily quadratically nonlinear Maxwell systems on cylindrical domains and demonstrate how to justify wavepacket approximations over long time scales within a class of admissible optical materials.

References

[1] Tomáš Dohnal, Mathias Ionescu-Tira and Marcus Waurick. Well-posedness and exponential stability of nonlinear Maxwell equations for dispersive materials with interface. *Journal of Differential Equations,* **383** (2024), pp. 24–77.

[2] Mathias Ionescu-Tira. Analysis, approximation and stability of electromagnetic surface waves. *Dissertation, MLU Halle*, 2023.

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MS07 **Wednesday, 17:00–17:25, SR 2.067**

Exponential Stability and Observabilty of the Maxwell System with Conductivity near the Boundary

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The following is joint work with Roland Schnaubelt (KIT).
We first study the anisotropic, linear Maxwell system on a bounded domain with conductivity σ in a collar around the boundary and perfectly conductive boundary conditions. Here we show that solutions with divergence free initial values decay exponentially to 0, using a Helmholtz decomposition of the solution and showing an observability estimate for the homogenous problem. For scalar coefficients and a uniformly positive conductivity this problem was considered in [1] and [2].
In the quasilinear setting we can also show exponential decay assuming isotropic coefficients and additional assumptions on $\nabla\sigma$. For anisotropic coefficients and uniformly positive σ and this was considered in [3].

References

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MS07 **Wednesday, 17:30–17:55, SR 2.067**

Local wellposedness in low regularity of Maxwell systems with retarded material laws on the full space

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In this talk we study the system of Maxwell equations in matter on $\mathbb{R}^3$. The response of the material to electromagnetic fields is described by a model commonly used in nonlinear optics which includes anisotropic and nonlinear retardations.

Local wellposedness in the space H^s with $s > 3/2$ has been proven by A. Babin and A. Figotin [1]. We show local wellposedness for mild solutions having H^s-regularity with $s \in (1, 3/2]$. The proof relies on splitting the fields into a charge-free part and a part involving charges, roughly speaking. The latter part is treated directly while the former is handled using Strichartz estimates for C^2-coefficients due to R. Schippa [2], see also [3]. This is joint work with R. Schnaubelt (Karlsruhe).

References

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MS07 **Wednesday, 18:00–18:25, SR 2.067**

An Optimal Control Framework for Maxwell's Equations

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This talk will focus on analytical and computational aspects related to dynamic optimization of electromagnetic phenomena. The optimization framework assumes the form of an optimal control problem for the Maxwell's system seeking a control strategy via distributed electric currents or boundary steering functions to optimize a prescribed objective functional under suitable constraints on the state and/or control variables. One of the main challenges lies in the complexity of the full time-dependent Maxwell's system which features a first-order hyperbolic coupled structure of semi- or quasilinear type. We will discuss how Pontryagin's and Bellman's principles can be used to treat this type of problems both analytically and numerically. If time permits, we will also showcase how these developments can be effectively combined with recent deep learning techniques to produce mathematically sound and computationally efficient ways to optimally steer electromagnetic systems.

MS07 **Thursday, 11:30–11:55, SR 2.067**

Finite energy solutions to quantum fluids models

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I will discuss the existence and stability of finite energy, weak solutions to a quantum Euler-Maxwell system, describing the dynamics of a charged quantum fluid in presence of quantized vortices [1]. The results will be obtained by exploiting the connection with a wave-function dynamics, governed by the nonlinear Maxwell-Schrödinger system. I will also discuss the persistence of higher order regularity [2,3].

References

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MS07	**Thursday, 12:00–12:25, SR 2.067**

Polychromatic Surface Waves in Kerr-Nonlinear Media

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A surface plasmon polariton (SPP) is an electromagnetic wave localized at the interface between two materials, typically a metal and a dielectric. The behavior of such waves is governed by Maxwell's equations, where the material functions are discontinuous across the interface.

In particular, we consider a non-linear and non-local in time dependence of the polarization on the electric field and search for a specific kind of solution, so called polychromatic solutions. These are inspired by the study of monochromatic waves, which are of the form

$$\mathcal{E}(x,t) = e^{-\mathrm{i}\omega t}E(x) + e^{\mathrm{i}\overline{\omega}t}\overline{E}(x)$$

and are used to determine a spectrum in the case of linear polarization. In the non-linear equation, however, such an ansatz may produce also terms of higher frequencies that cannot be compensated by the low frequency modes.

We thus introduce an ansatz as a series where every integer multiple of ω appears as a frequency. Maxwell's equations then decompose into a series of weakly coupled, linear equations. We show that such an ansatz produces solutions to Maxwell's equations an discuss their properties.

MS07	**Thursday, 12:30–12:55, SR 2.067**

Existence of breather solutions to Maxwell's equations

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We consider the wave equation

$$-u_{xx} - V(x)u_{tt} = \Gamma(x)N(u)_{tt}$$

for $u\colon \mathbb{R}\times\mathbb{R}\to\mathbb{R}$ and a cubic nonlinearity N, which arises as an exact reduction of Maxwell's equations for Kerr-type optical materials.

We show existence and regularity of time-periodic, real-valued, localized solutions (called breathers) under strong assumptions on the potentials V, Γ and the nonlinearity N. In particular, we assume that the linear operator $-\partial_x^2 - V(x)\partial_t^2$ is elliptic and that the nonlinearity is given by $N(u)(x,t) = \left(\int_0^\infty \kappa(\tau)u(x,t-\tau)\,\mathrm{d}\tau\right)^3$, allowing us to find breathers variationally using the mountain-pass method.

We also consider hyperbolic linear operators $-\partial_x^2 - V(x)\partial_t^2$ where 0 lies in a spectral gap. Here we search for breathers on the generalized Nehari manifold.

MS07 **Thursday, 16:30–16:55, SR 2.067**

Travelling waves for Maxwell's equations in nonlinear and symmetric media

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We look for travelling wave fields

$$E(x, y, z, t) = U(x, y) \cos(kz + \omega t) + \widetilde{U}(x, y) \sin(kz + \omega t), \quad (x, y, z) \in \mathbb{R}^3,\ t \in \mathbb{R},$$

satisfying Maxwell's equations in a nonlinear and cylindrically symmetric medium. We obtain a sequence of solutions with diverging energy that is different from that obtained by McLeod, Stuart, and Troy in 1992. the obtained solutions are the so-called TM-modes. In addition, we consider a more general nonlinearity, controlled by an N-function.

MS07 **Thursday, 17:00–17:25, SR 2.067**

Semiclassical states for the curl-curl problem

Bartosz Bieganowski[1,*]
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We show the existence of the so-called semiclassical states $\mathbf{U} : \mathbb{R}^3 \to \mathbb{R}^3$ to the following curl-curl problem

$$\varepsilon^2 \nabla \times (\nabla \times \mathbf{U}) + V(x)\mathbf{U} = g(\mathbf{U}),$$

for sufficiently small $\varepsilon > 0$. We study the asymptotic behaviour of solutions as $\varepsilon \to 0^+$ and we investigate also a related nonlinear Schrödinger equation involving a singular potential. The problem models large permeability nonlinear materials satisfying the system of Maxwell equations.

This is a joint work with Adam Konysz and Jarosław Mederski.

References

[1] B. Bieganowski, A. Konysz, and J. Mederski, Semiclassical states for the curl-curl problem, *arXiv:2312.03658*

MS07 Thursday, 17:30–17:55, SR 2.067

Dual variational methods for Nonlinear Maxwell Equations

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In this talk we discuss a nonstandard variational approach to nonlinear Maxwell's equations on Euclidean space that allows one to prove the existence of solutions in the case of asymptotically vanishing material parameters. We point out a number of open problems that currently prevent us from studying more general models.

References

[1] R. Mandel, A simple variational approach to Nonlinear Maxwell equations, *arXiv:2109.00826v3*.

Minisymposium 08

Discretizations of Nonlinear and Multiscale Wave-Type Problems

Organizers: Benjamin Dörich, Patrick Henning, and Roland Maier

MS08	Wednesday, 15:00–15:25, SR 0.014

Super-localized Orthogonal Decomposition for high-frequency Helmholtz problems

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We propose a novel variant of the Localized Orthogonal Decomposition (LOD) method for time-harmonic scattering problems of Helmholtz type with high wavenumber κ. On a coarse mesh of width H, the proposed method identifies local finite element source terms that yield rapidly decaying responses under the solution operator. They can be constructed to high accuracy from independent local snapshot solutions on patches of width ℓH and are used as problem-adapted basis functions in the method. In contrast to the classical LOD and other state-of-the-art multi-scale methods, the localization error decays super-exponentially as the oversampling parameter ℓ is increased. This implies that optimal convergence is observed under the substantially relaxed oversampling condition $\ell \gtrsim (\log \frac{\kappa}{H})^{(d-1)/d}$ with d denoting the spatial dimension. Numerical experiments demonstrate the significantly improved offline and online performance of the method also in the case of heterogeneous media and perfectly matched layers.

References

[1] P. Freese, M. Hauck, D. Peterseim, Super-Localized Orthogonal Decomposition for High-Frequency Helmholtz Problems, *SIAM Journal on Scientific Computing* **46**(4):A2377–A2397 (2024).

MS08 **Wednesday, 15:30–15:55, SR 0.014**

A Galerkin Method with Microlocalised Shape Functions to Solve High-frequency Helmholtz Problems

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High-frequency Helmholtz problems are very challenging to solve with the Finite Element Method. Indeed, the number of degrees of freedom needed to correctly approximate the solution grows at least as k^d, where k is the wavenumber and d the dimension.

In [1], it has been proposed to use the Galerkin or the Least Squares method with *Gaussian coherent states* (GCS) as basis functions. For a wide range of source terms, the Least Squares formulation with GCS provides an accurate solution with only about $k^{d-\frac{1}{2}}$ degrees of freedom. However, no similar result has been proved for the Galerkin formulation, and it has been observed that GCS can lead to ill-conditioned discretisation matrices.

In this work, we use as basis functions particular linear combinations of GCS which lead to well-conditioned discretised problems. We will show that with these new shape functions, both the Galerkin and the Least Squares formulations provide an accurate solution with about $k^{d-\frac{1}{2}}$ degrees of freedom. We will illustrate these results with 1D numerical examples.

References

[1] T. Chaumont-Frelet, V. Dolean and M. Ingremeau, Efficient approximation of high-frequency Helmholtz solutions by Gaussian coherent states, *Numerische Mathematik*, 2024, vol. 156, no 4, pp. 1385-1426.

MS08 **Wednesday, 16:30–16:55, SR 0.014**

Multiscale methods for wave propagation in high-contrast media

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To obtain enhance or even unusual wave properties, metamaterials typically do not only use fine-scale structures, but also high contrast between the different materials. For instance, subwavelength structures can thereby induce resonances at moderate frequencies or wave lengths. For time-harmonic problems such as the Helmholtz equation, homogenization results as well as numerical multiscale methods are quite well developed, see [1] for an overview.

However, when considering time-dependent problems such as the wave equation, new phenomena seem to occur and new challenges arise for the numerical discretization and its error analysis. At the example of the Localized Orthogonal Decomposition as numerical multiscale method, we will discuss the design and the error estimates for the wave equation with high-contrast coefficients based on [2]. Comparisons with the time-harmonic case will be made and numerical experiments will illustrate the theoretical findings.

This talk reports on joint work with Élise Fressart.

References

[1] B. Verfürth, Numerical multiscale methods for waves in high-contrast media, *Jahresber. Dtsch. Math.-Ver.* **126** (2024), pp. 37–65.

[2] É. Fressart and B. Verfürth, Wave propagation in high-contrast media: periodic and beyond, *Comput. Methods Appl. Math.* **24** (2024), pp. 337—354.

MS08 **Wednesday, 17:00–17:25, SR 0.014**

Homogenization of time-varying wave-type problems

Christian Döding[1,*], **Barbara Verfürth**[1]
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In this talk we present recent developments in the analytical and numerical homogenization of time-varying multiscale problems. Such problems often arise in contexts of light propagation through so-called metamaterials, where material properties fluctuate on very small time scales. The underlying mathematical model is a wave-type equation with time-varying multiscale coefficients, derived as a simplification of Maxwell's equations. We will exploit both the theoretical framework and the computational techniques used to address these challenges, focusing on how homogenization techniques can effectively bridge the gap between microscale dynamics and macroscopic behavior.

MS08 **Wednesday, 17:30–17:55, SR 0.014**

A higher-order multiscale method for the heterogeneous wave equation

Balaje Kalyanaraman[1], **Felix Krumbiegel**[2,*], **Roland Maier**[2], **Siyang Wang**[1]
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In this talk we apply the higher-order extension of the localized orthogonal decomposition method to the heterogeneous wave equation. The method corrects coarse-scale classical piecewise polynomial basis functions. These basis functions are suboptimal when considering time-dependent problems, which we remedy by introducing additional corrections. Combined with a suitable higher-order time discretization they result in an arbitrary higher order scheme in space and time, even under minimal regularity assumptions in space. We conclude the talk with numerical examples which confirm the theoretical findings and hint on possible localization strategies for the additional corrections.

MS08 **Wednesday, 18:00–18:25, SR 0.014**

A Multiscale Method for Helmholtz Based on the Temporal Wave Equation

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[2]*Department of Mathematics, KTH, Stockholm, Sweden*

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We are interested in finding solutions to the Helmholtz equation with rapidly varying coefficients. For these problems, direct discretizations must resolve the smallest spatial scale, which may be prohibitively costly. Instead we design a multiscale method. We first rewrite the Helmholtz equation as the limit of iterations of certain filtered time-domain solutions, so-called WaveHoltz iterations. In this way we can apply existing HMM multiscale methods for the wave equation in the iterations. Moreover, we can use the wave equation on the micro level, which greatly simplifies the boundary condition issue in the micro problems. Another benefit of using WaveHoltz iterations comes when considering large scale problems. Implementing parallelized, high order methods that run efficiently on big computers is typically easier to do for time domain methods than for traditional Helmholtz solvers. This advantage carry over to the multiscale method presented here.

MS08 **Thursday, 11:30–11:55, SR 1.067**

Dynamic Ritz projection of finite element methods for fluid-structure interaction

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Regardless of the development of various finite element methods for fluid-structure interaction (FSI) problems, optimal-order convergence of finite element discretizations of the FSI problems in the $L^\infty(0,T;L^2)$ norm has not been proved due to the incompatibility between standard Ritz projections and the interface conditions in the FSI problems. To address this issue, we define a dynamic Ritz projection (which satisfies a dynamic interface condition) associated to the FSI problem and study its approximation properties in the $L^\infty(0,T;H^1)$ and $L^\infty(0,T;L^2)$ norms. Existence and uniqueness of the dynamic Ritz projection of the solution, as well as estimates of the error between the solution and its dynamic Ritz projection, are established. By utilizing the established results, we prove optimal-order convergence of finite element methods for the FSI problem in the $L^\infty(0,T;L^2)$ norm.

MS08 Thursday, 12:00–12:25, SR 1.067

A Gonzalez scheme for smooth nonlinear contact conditions in time-domain wave propagation problems

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We consider the problem of transient wave propagation within two domains linked with smooth nonlinear contact conditions at a common interface. While standard linear elastodynamics is assumed within each domain, at the interface we consider continuity of normal stresses, and (more importantly) a smooth finite compressibility law

$$\boldsymbol{\sigma}_2 \cdot \boldsymbol{n} = \boldsymbol{\sigma}_1 \cdot \boldsymbol{n} = \boldsymbol{\sigma} \cdot \boldsymbol{n}, \quad \boldsymbol{\sigma} \cdot \boldsymbol{n} = K_N \mathcal{R}([\boldsymbol{u}] \cdot \boldsymbol{n})\boldsymbol{n} + K_T([\boldsymbol{u}] \cdot \boldsymbol{t})\boldsymbol{t},$$

where $\boldsymbol{n}$ and $\boldsymbol{t}$ are the normal and tangent vector fields of the interface, $[\boldsymbol{u}] \cdot \boldsymbol{n} = (\boldsymbol{u}_2 - \boldsymbol{u}_1) \cdot \boldsymbol{n}$ is the "crack opening displacement"[1], K_N and K_T are stiffness parameters, while $\mathcal{R}(\cdot)$ is a smooth strictly increasing function penalizing the compression regime $[\boldsymbol{u}] \cdot \boldsymbol{n} < 0$. We propose an energy preserving – thus stable – time scheme based upon [2], and devise an efficient time-marching algorithm. We validate our approach with semi-analytical results, and illustrate typical nonlinear waves phenomena (harmonics, zero-frequency components) in 2D.

References

[1] J. D. Achenbach and A. N. Norris, Loss of specular reflection due to nonlinear crack-face interaction, J. Nondestruct. Eval., 1982, 3, 229-239.

[2] O. Gonzalez, Exact energy and momentum conserving algorithms for general models in nonlinear elasticity, Comput. Methods Appl. Mech. Eng., 2000, 190(13-14), 1763-1783.

MS08 Thursday, 12:30–12:55, SR 1.067

Numerical Analysis for Electromagnetic Scattering with Nonlinear Boundary Condistion

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The talk covers a time-dependent electromagneticscattering problem from obstacles whose interaction with the wave is fully determined by a nonlinear boundary condition. In particular, the boundary condition studied enforces a power law type relation between the electric and magnetic field along the boundary. Based on time-dependent jump conditions of classical boundary operators, we derive a nonlinear system of time-dependent boundary integral equations that determines the tangential traces of the scattered electric and magnetic fields.

Fully discrete schemes are obtained by discretising the nonlinear boundary integral equations with Runge–Kutta based convolution quadrature in time and Raviart–Thomas boundary elements in space. Error bounds with explicitly stated convergence rates are presented. Numerical experiments illustrate the use of the proposed method and provide empirical convergence rates.

MS08 **Thursday, 15:00–15:25, SR 1.067**

Filtered finite difference methods for nonlinear Schrödinger equations in semiclassical scaling

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We introduce a new numerical strategy for solving semiclassically scaled nonlinear Schrödinger equations with highly oscillatory initial data, without the need to resolve high-frequency oscillations in both space and time by prohibitively fine grids as would be required by standard finite difference methods. Our approach modifies traditional finite difference methods by incorporating appropriate filters. Specifically, we propose the filtered leapfrog and filtered Crank–Nicolson methods, both of which achieve second-order accuracy with time step τ and mesh size h that are not restricted by the small semiclassical parameter. Furthermore, the filtered Crank–Nicolson method conserves both the discrete mass and a discrete energy. Numerical experiments illustrate the theoretical results. The talk is based on joint work with Yanyan Shi.

MS08 **Thursday, 15:30–15:55, SR 1.067**

Super-localized wave function approximation of Bose-Einstein condensates

Johan Wärnegård[1,*], Daniel Peterseim [2], Christoph Zimmer[2]

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[2] *Institute of Mathematics, Augsburg University, Germany*

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In this talk I will discuss how the method of super localized orthogonal decomposition can be used to solve the cubic nonlinear Schrödinger equation efficiently. By adapting the FEM shape functions to the partial differential operator associated with the problem (e.g. a suitable linearization of the full nonlinear operator), favorable optimal convergence rates known for smooth solutions are preserved in problems of low regularity due to the presence of rough potentials . In addition, a quadrature-like simplification is introduced that dramatically speeds up the assembly of nonlinear terms, without any loss of accuracy even for low regularity potentials. For smooth problems, the approach results in a spline space in which an extra order of convergence can be obtained on structured grids by means of a gradient recovery scheme. The talk is largely based on the paper [1]. To demonstrate the competitiveness of the method, some computationally challenging ground states and dynamics of Bose-Einstein condensates will be showcased.

References

[1] D. Peterseim, J. Wärnegård, C. Zimmer *Journal of Computational Physics* **510** (2024), pp. 113097.

MS08 **Thursday, 16:30–16:55, SR 1.067**

An implicit-explicit time discretization scheme for second-order semilinear damped wave equations with application to multiscale problems

Daniel Eckhardt[1,*], **Marlis Hochbruck**[1], **Barbara Verfürth**[2]

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In this talk we present an implicit-explicit scheme (IMEX) for semilinear damped wave equations. By treating the nonlinear term explicitly and the linear, stiff part implicitly, we obtain a method which is not only unconditionally stable but also highly efficient. Our main results are error bounds of the full discretization in space and time for the IMEX scheme combined with a general abstract nonconforming space discretization [1]. As an application, we consider the Heterogeneous Multiscale Method for wave equations with coefficients highly oscillating in space.

References

[1] D. Eckhardt, M. Hochbruck, B. Verfürth, Error analysis of an implicit-explicit time discretization scheme for semilinear wave equations with application to multiscale problems, *CRC1173 Preprint* 2024/13.

MS08 **Thursday, 17:00–17:25, SR 1.067**

Time integration of oscillatory nonlinear Dirac equations with nonresonant step-sizes

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In the nonrelativistic limit, solutions of nonlinear Dirac equations oscillate in time with high frequency. As a consequence, standard time integrators only provide accurate approximations if the step-size τ is so small that the oscillations are resolved by many time steps. We present a tailor-made time integrator which does not suffer from such a step-size restriction. The basic principle is to apply Duhamel's formula twice, to approximate slowly varying parts of the integrands, and to compute the remaining highly oscillatory integrals exactly. Instead of aiming for a local error of $O(\tau^3)$, however, we also omit those terms of $O(\tau^2)$ which involve certain highly oscillatory phases. This simplifies the method considerably and does not spoil the accuracy significantly if a nonresonant step-size is chosen, because then the corresponding local error terms cancel in the error accumulation. We present and discuss a rigorous error bound and illustrate it by numerical examples.

MS08 **Thursday, 17:30–17:55, SR 1.067**

Time-splitting schemes for the cold-plasma model using Finite Element Exterior Calculus

Elena Moral-Sánchez[1,*], Martin Campos-Pinto[1], Yaman Güçlü[1], Omar Maj[1]
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The cold-plasma model is used to study the propagation of electromagnetic waves in a magnetized plasma, which is of special interest for reflectometry and plasma wave heating in magnetic-confinement fusion devices [1]. In this talk we present a finite element space discretization relying on isogeometric B-splines, which preserves the de Rham structure of Maxwell's equations and the Hamiltonian structure of the cold-plasma model. We use Silver-Müller boundary conditions to prescribe an oscillatory boundary source. To solve the resulting non-autonomous system, we propose two geometric time-splitting integrators [2] which preserve either the Poisson matrix or the Hamiltonian. In addition to studying their accuracy and stability properties, we compare their performance. Finally, we present simulations of complex wave propagation in heterogeneous media corresponding to turbulent plasma densities.

References

[1] S. Heuraux et al., Study of wave propagation in various kinds of plasmas using adapted simulation methods, with illustrations on possible future applications, *Comptes Rendus Physique* **15**, 5 (2014), pp. 421–429.

[2] E. Hairer and G. Wanner and C. Lubich, *Geometric Numerical Integration: Structure Preserving Algorithms for Ordinary Differential Equations*, Springer Berlin, Heidelberg, 2006.

Minisymposium 09

Water Waves and Dispersive Equations

Organizers: Mats Ehrnström and Douglas Svensson Seth

MS09	Tuesday, 11:30–11:55, SR -1.025

Why and how to rectify a model for surface gravity waves

Vincent Duchene[1,*], **Benjamin Melinand**[2]
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We will discuss the shortcomings of the following system of equations modeling the dynamical evolution of small-slope surface gravity waves over an infinite-depth layer of water.

$$\begin{cases} \partial_t\zeta - |D|\psi + \varepsilon|D|(\zeta|D|\psi) + \varepsilon\nabla_{\mathbf{x}}\cdot(\zeta\nabla_{\mathbf{x}}\psi) = 0, \\ \partial_t\psi + \zeta + \frac{\varepsilon}{2}\left(|\nabla_{\mathbf{x}}\psi|^2 - (|D|\psi)^2\right) = 0, \end{cases}$$

where $|D| = (-\Delta_{\mathbf{x}})^{1/2}$, $\mathbf{x} \in \mathbb{R}^d$ or $\mathbf{x} \in \mathbb{T}^d$, $d \in \{1, 2\}$.

We will exhibit the plausible source of high-frequency instabilities that are observed in numerical simulations. Once this is done, we will propose a modification of the equations which (i) is costless from the point of view of numerical discretization, (ii) preserves the Hamiltonian structure of the system, (iii) does not harm the precision of the model, and (iv) restores large time well-posedness and eventually full justification with respect to the full water waves model.

This talk is based on a work with Benjamin Melinand [1].

References

[1] V. Duchêne and B. Melinand, Rectification of a deep water model for surface gravity waves, *Pure Appl. Anal.* **6**(1) (2024), pp. 73–128.

MS09 **Tuesday, 12:00–12:25, SR -1.025**

Numerical study of Boussinesq systems

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We present a numerical study of solutions to Boussinesq systems appearing in the theory of surface waves. We study integrable and non-integrable examples. Solitary waves in 1D are constructed, and their stability is studied numerically. The time evolution of localised initial data is explored. Of special interest is the role of the non-cavitation condition. The appearence of dispersive shock waves, zones of rapidly modulated oscillations, in the vicinity of shocks of the corresponding dispersionless systems is studied.

References

[1] C. Klein, J.-C. Saut, Numerical study of the Amick-Schonbek system, *Stud Appl Math.* 2024;1-22. https://doi.org/10.1111/sapm.12691

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MS09 **Tuesday, 12:30–12:55, SR -1.025**

Rigorous analysis of spectral schemes for systems of Whitham–Boussinesq type

Vincent Duchêne[1], **Johanna Marstrander**[2,*]
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Numerical simulations of the evolution of surface gravity waves in Whitham–Boussinesq-type models typically employ pseudospectral schemes. Considering finite Fourier sums and using perfect dealiasing, the spatial discretization can be written explicitly as a PDE with the Fourier multiplier P_N acting as a low-pass filter for wavenumbers $|\boldsymbol{k}| \leq N$:

$$\begin{cases} \partial_t \zeta_N + \mathsf{F}_1^\mu \nabla \cdot \boldsymbol{v}_N + \mathsf{P}_N \mathsf{F}_2^\mu \nabla \cdot (\zeta \mathsf{F}_2^\mu \boldsymbol{v}_N) = 0, \\ \partial_t \boldsymbol{v}_N + \nabla \zeta + \mathsf{P}_N((\mathsf{F}_2^\mu \boldsymbol{v}_N \cdot \nabla) \mathsf{F}_2^\mu \boldsymbol{v}_N) = 0, \end{cases}$$

For example, setting $F_1^\mu = F_2^\mu = \frac{\tanh(\sqrt{\mu}|D|)}{\sqrt{\mu}|D|}$ corresponds to the Whitham-Boussinesq system introduced in [1]. Using energy estimates, we investigate the stability and convergence of the approximate solution as $N \to \infty$. The results depend on the regularity of the initial data and whether the system is symmetric or symmetrizable, semilinear or quasilinear. The talk is based on joint work with Vincent Duchêne.

References

[1] E. Dinvay, D. Dutykh and H. Kalisch, A comparative study of bi-directional systems. *Appl. Numer. Math.*, **141** (2019), pp. 248-–262.

MS09 **Tuesday, 15:00–15:25, SR -1.025**

Properties of traveling wave solutions to nonlinear dispersive equations

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We will talk about properties of traveling wave solutions to nonlinear dispersive equations. In particular, we consider the symmetry of traveling waves at the presence of weak dispersion. In addition, the relation between symmetry and the steady properties of solutions will be introduced. This talk is based on collaboration with Gabriele Bruell, Anna Geyer, Mats Ehrnstrom, Fengyang Xiao and Pan Zhang.

MS09 **Tuesday, 15:30–15:55, SR -1.025**

Asymmetric travelling wave solutions of the capillary–gravity Whitham equation

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By a bifurcation argument we prove that the capillary-gravity Whitham equation features *asymmetrical* periodic travelling wave solutions of arbitrarily small amplitude. Such waves exist only in the weak surface tension regime $0 < T < \frac{1}{3}$ and are necessarily bimodal; they are located at double bifurcation points satisfying a certain symmetry breaking condition. Our bifurcation argument is an extension of the one applied by Ehrnström et al. to find symmetric waves: Here, two additional scalar equations arise. Combining the variational structure of our problem with its translation symmetry, we show that these two additional equations are in fact linearly dependent, and can (at 'symmetry breaking' bifurcation points) be solved by incorporating the surface tension as a bifurcation parameter. Contrary to the symmetric case, only very specific modal pairs (k_1, k_2) give rise to (small) asymmetrical periodic waves and we here provide a partial characterization of such pairs.

References

[1] Mats Ehrnström and Mathew A. Johnson and Ola I. H. Maehlen and Filippo Remonato, On the Bifurcation Diagram of the Capillary–Gravity Whitham Equation, *Water Waves* **2** (2019), pp. 275–313.

MS09 **Tuesday, 16:30–16:55, SR -1.025**

A resonant Lyapunov centre theorem with an application to hydroelastic waves

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We will present a generalized Lyapunov centre theorem for an antisymplectically reversible Hamiltonian system with a nondegenerate 1:1 or 1:-1 semisimple resonance. In addition we allow for a non-constant symplectic structure and that the origin is in the spectrum of the linearized Hamiltonian vector field. As an application we will show how this theorem can be used to prove existence of doubly periodic hydroelastic waves by first formulating the hydrodynamic problem as a reversible Hamiltonian system, using Kirchgässner's spatial dynamics approach.

MS09 **Tuesday, 17:00–17:25, SR -1.025**

Global Bifurcation of Three-Dimensional Gravity-Capillary Waves on Beltrami Flows

Bastian Hilder[1], Erik Wahlén[2], Giang To[2]
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Most of the current theory on three-dimensional steady water waves assumes irrotational flow. One exception is the construction of a family of small-amplitude doubly periodic gravity-capillary waves on Beltrami flows by Erik Wahlén, Lokharu and Svensson Seth from 2020. In my talk I will describe a global continuation of this family. One of the challenges is that the local family is constructed using a multiparameter bifurcation approach, whereas global bifurcation theory usually assumes a single bifurcation parameter. Our theory includes irrotational flow as special case.

This is joint work with Bastian Hilder and Erik Wahlén.

MS09 **Tuesday, 17:30–17:55, SR -1.025**

Symmetric doubly periodic gravity-capillary waves with small vorticity

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Earlier efforts for rotational three-dimensional water waves have focused on the case of a Beltrami velocity field $\mathbf{u}$. That is, where $\text{curl}(\mathbf{u})$ is colinear with $\mathbf{u}$. Inspired by an approach used by Lortz [1] in the context of magnetohydrostatics, we construct small symmetric doubly-periodic gravity-capillary waves with a non-Beltrami velocity field.

References

[1] D. Lortz, Über die Existenz toroidaler magnetohydrostatischer Gleichgewichte ohne Rotationstransformation, *Zeitschrift für angewandte Mathematik und Physik* **21** (1970), pp. 196-211

MS09 **Tuesday, 18:00–18:25, SR -1.025**

On bounds for the Froude number for solitary water waves

Jörg Weber[1,*], **Evgeniy Lokharu**[2]

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A classical and central problem in the theory of water waves is to classify parameter regimes for which nontrivial solitary waves can and do exist. In the two-dimensional, irrotational, pure gravity case, the non-dimensional Froude number $F = c/\sqrt{gd}$ (with c the wave speed, g the gravitational constant, and d the depth of the fluid at infinity) plays the central role. So far it is known that necessarily $F \in (1, \sqrt{2})$ for a nontrivial solitary wave. While the lower bound is sharp, numerics indicate that the upper bound is *not* sharp. In this talk, we discuss a new strategy utilising the flow force function and rigorously establish an improved upper bound for F. Moreover, we outline similar results in the case of constant vorticity.

MS09 **Wednesday, 15:00–15:25, SR 3.068**

On the collision of two solitary waves of the Zakharov–Kuznetsov equation

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The Zakharov-Kuznetsov (ZK) equation in dimension 2 is a generalization in plasma physics of the one-dimensional Korteweg de Vries equation (KdV). Both equations admit solitary waves, that are solutions moving in one direction at a constant velocity, vanishing at infinity in space. When two solitary waves collide, two phenomena can occur: either the structure of two solitary waves is conserved without any loss of energy and change of sizes (elastic collision), or the structure is lost or modified (inelastic collision). As a completely integrable equation, KdV only admits elastic collisions. In this talk, we describe the collision of two solitary waves having almost same size for ZK on the whole time interval. We also give a stability result of this phenomenon and approach the question of inelasticity.

References

[1] D. Pilod and F. Valet, Asymptotic stability of a finite sum of solitary waves for the Zakharov-Kuznetsov equation, *Nonlinearity* **37** (2024), p 41.

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MS09 **Wednesday, 15:30–15:55, SR 3.068**

Traveling wave solutions of the fractional Kadomtsev–Petviashvili equation

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The fractional Kadomtsev–Petviashvili (fKP) equation has two versions, fKP-I and fKP-II, as the classical KP equation. We discuss the existence and nonexistence results for the traveling wave solutions of the fKP equation for both versions. We also conduct some numerical experiments to obtain these solutions and investigate their dynamical properties.

Minisymposium 10

Bifurcation and Pattern Formation in Dynamical Systems

Organizers: André H. Erhardt, Bastian Hilder, and Jörg Weber

MS10	Monday, 11:45–12:10, SR 2.066

Chaotic Front Motion in Allen-Cahn Equations with Large Scale Linear Fields

Jens D.M. Rademacher[1,*], **Martina Chirilus-Bruckner**[2], **Peter van Heijster**[3]
[1] *University of Hamburg, Department of Mathematics, Hamburg, D*
[2] *University of Leiden, Department of Mathematics, Leiden, NL*
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It is well known that front interfaces in bistable isotropic reaction-diffusion equations are asymptotically stationary or move with constant speed. It turns out that self-organised front motion by interacting components occurs in multi-component reaction-diffusion system with one fast component governed by an Allen-Cahn equation that is coupled to slow linear equations. We consider N additional components and study existence, stability and bifurcations of stationary and uniformly travelling front solutions near the singular limit. Combining an Evans function and a functional analytic approach we find that the dynamics of front speeds obeys an N-th order ODE. Analysing the normal form, for $N = 3$ we prove that chaotic dynamics occurs in the sense of an unfolded Shil'nikov homoclinic orbit. Numerical study guided by our analytic findings complement these results.

This is joint work with Martina Chirilus-Bruckner (Leiden) and Peter van Heijster (Wageningen).

MS10 **Monday, 12:15–12:40, SR 2.066**

Selection of interface angles through directional quenching – an Allen-Cahn system

Olivia Clifton (Cannon)[1,*]

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One motivation for studying Allen-Cahn systems is that evolution of interfaces approximates mean-curvature flow. We consider two competing effects on interfaces in a planar Allen-Cahn system, with quenching in the right half-plane: first, asymmetry in the nonlinearity, which corresponds to driven mean-curvature flow; and second, translation of the quenching line. We find, for small speeds and asymmetries, that the quenching speed uniquely determines the farfield angle of a solution with asymptotically straight interface. This result can be interpreted as finding the effective boundary condition from the quenching line for the mean-curvature flow in the fairfield.

MS10 **Monday, 12:45–13:10, SR 2.066**

On the Existence of Fully Localised Planar Patterns

Mark D. Groves[1], **Dan J. Hill**[1,*]

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Spatially localised patterns are known to emerge in a variety of physical settings, ranging from dryland vegetation to vibrating fluids to the buckling of cylinders. While there are a number of mathematical tools for studying localised patterns in one spatial dimension, developing equivalent approaches in higher spatial dimensions remains a major challenge in the area of pattern formation. In this talk, we will focus on 2D patterns that are localised in the radial direction and present a novel formal approach to derive radial amplitude equations [1], which provides new insight into the emergence of fully localised planar patterns. As an example, we derive radial amplitude equations for fully localised hexagons and quasipatterns in the Swift–Hohenberg equation (SHE), for which we can obtain explicit localised solutions. We then present an existence proof for fully localised stripes in the SHE, using modulation theory from water waves [2,3] extended to polar coordinates. To the authors' knowledge, this provides the first existence proof of radially-localised non-axisymmetric planar patterns.

References

[1] D.J. Hill and D.J.B. Lloyd, Radial amplitude equations for fully localised planar patterns, arXiv preprint (2024), arXiv:2403.02949v2

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[3] M. Ehrnström and M.D. Groves, Small-amplitude fully localised solitary waves for the full-dispersion Kadomtsev–Petviashvili equation, *Nonlinearity* **31** (2018)

MS10 **Monday, 15:30–15:55, SR 2.066**

On the vorticity threshold for steady water waves

Evgeniy Lokharu[1,*], **Miles Wheeler**[2]

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In this talk we will discuss effects of a strong adverse constant vorticity on the global dynamics of Stokes and solitary waves. We will prove that for constant vorticities above some threshold value the horizontal velocity on the surface is always separated from zero by a constant depending only on the vorticity. In particular this forbids formation of extreme and overhanging waves before a critical layer appears inside of the fluid, first at the bottom right below the crest. This also shows that the slopes of unidirectional solutions are uniformly bounded by a small constant, provided the vorticity is large enough and is non-favorable. We also obtain a non-trivial bound for the amplitude. Our analytic results agree with and confirm the numerical analysis by Ko and Strauss (2008).

If you have further questions, please do not hesitate to contact us at

`evgeniy.lokharu@lund.math.se, mw2319@bath.ac.uk`

MS10 **Monday, 16:30–16:55, SR 2.066**

From Turing instability to large periodic patterns

Gianne Derks[1], **Christopher Brown**[2], **Peter van Heijster**[3], **David J.B. Lloyd**[2]

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Following pattern formation away from onset is still a challenge as no general theory is available. In this talk, we consider a three component reaction-diffusion system with one of its components singularly perturbed, this component is known as the fast variable. We start with a numerical investigation to motivate and illustrate the development of an analytical theory describing the periodic patterns emerging from a Turing instability using geometric singular perturbation theory. We show analytically that after the initial Turing instability, spatially periodic patterns evolve into a small amplitude spike in the fast variable whose amplitude grows as one moves away from onset. This is followed by a secondary transition where the spike in the fast variable widens, its periodic pattern develops two sharp transitions between two flat states and the amplitudes of the other variables grow. The final type of transition we uncover analytically is where the flat states of the fast variable develop structure in the periodic pattern. We conclude with a preliminary numerical investigation where we uncover more complicated periodic patterns and snaking-like behaviour that are driven by the three transitions analysed in this paper.

MS10 **Monday, 17:00–17:25, SR 2.066**

Global existence for long wave Hopf unstable spatially extended systems with a conservation law

Nicole Gauss[1], **Anna Logioti**[1,*], **Guido Schneider**[1], **Dominik Zimmermann**[1]

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We are interested in reaction-diffusion systems, with a conservation law, exhibiting a Hopf bifurcation at the spatial wave number $k = 0$. With the help of a multiple scaling perturbation ansatz a Ginzburg-Landau equation coupled to a scalar conservation law can be derived as an amplitude system for the approximate description of the dynamics of the original reaction-diffusion system near the first instability. We use the amplitude system to show the global existence of all solutions starting in a small neighborhood of the weakly unstable ground state for original systems posed on a large spatial interval with periodic boundary conditions.

MS10	**Monday, 17:30–17:55, SR 2.066**

Well-posedness of a semilinear parabolic equation arising from the modelling of atmospheric flows

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Recently, Constantin and Johnson [1] derived a model describing *morning glory clouds*, a spectacular atmospheric phenomenon whereby elongated tubular clouds travel perpendicularly to the cloud line, in an essentially two-dimensional motion. Exploiting conservation of mass to eliminate the vertical velocity component, the problem can be reduced to studying a semilinear parabolic equation with nonlocal nonlinearity. The goal of this talk is to illustrate some recent results concerning the well-posedness of this problem, including local strong well-posedness and existence of global weak solutions [2] as well as global strong well-posedness for small initial data [3].

References

[1] A. Constantin and R. S. Johnson, On the propagation of nonlinear waves in the atmosphere, *Proc. R. Soc. A* **478** (2022), no. 20210895.

[2] B.-V. Matioc and L. Roberti, Weak and classical solutions to an asymptotic model for atmospheric flows, *J. Differ. Equ.* **367** (2023), pp. 603–624.

[3] B.-V. Matioc, L. Roberti and C. Walker, Quasilinear parabolic equations with superlinear non-linearities in critical spaces, *preprint*, arXiv:2408.05067.

MS10	**Tuesday, 15:00–15:25, SR 0.014**

Geometric Singular Perturbation Theory for PDEs

Christian Kuehn[1,*], **Maximilian Engel, Felix Hummel, Samuel Jelbart, Pascal Lehner, Nikola Popovic, Mariya Ptashnyk, Jan-Eric Sulzbach, Thomas Zacharis**

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In this talk, I am going to report on recent progress in [1-8] on lifting techniques from GSPT to infinite-dimensional PDEs. The main focus will on generalizing the blow-up method to desingularize non-hyperbolic steady states.

References

[1] "Fast reactions and slow manifolds", C. Kuehn and J.-E. Sulzbach, arXiv

[2] "Geometric analysis of fast-slow PDEs with fold singularities", M. Engel, F. Hummel, C. Kuehn, N. Popović, M. Ptashnyk and T. Zacharis, arXiv

[3] "Geometric blow-up of a dynamic Turing instability in the Swift-Hohenberg equation", F. Hummel, S. Jelbart and C. Kuehn, arXiv

[4] "Blow-up analysis of fast-slow PDEs with loss of hyperbolicity", M. Engel and C. Kuehn, arXiv

[5] "Infinite dimensional slow manifolds for a linear fast-reaction systems", C. Kuehn, P. Lehner and J.-E. Sulzbach, Contemporary Mathematics, AMS, accepted / to appear, 2023

[6] "A formal geometric blow-up method for pattern forming systems", S. Jelbart and C. Kuehn, Contemporary Mathematics, AMS, accepted / to appear, 2023

[7] "Slow manifolds for infinite-dimensional evolution equations", F. Hummel and C. Kuehn, Commentarii Mathematici Helvetici, Vol. 97, No. 1, pp. 61-132, 2022

[8] "Connecting a direct and a Galerkin approach to slow manifolds in infinite dimensions", M. Engel, F. Hummel and C. Kuehn, Proceedings of the American Mathematical Society, Vol. 8, pp. 252-266, 2021

MS10 **Tuesday, 15:30–15:55, SR 0.014**

Spatial heterogeneity and striped phases

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I will discuss the effect of spatial inhomogeneities on striped phases, focusing mostly on spatially localized disturbances but as time permits mention spatial gradients. Technically, the problem here is to describe a "spatial center manifold" at spatial infinity and asymptotics of solutions within this manifold, in particular when spatial infinity is higher-dimensional. Our results describe these dynamics in terms of multipole expansions but also point to a more general algebra for these infinite-dimensional asymptotics.

Minisymposium 11

Monotonicity and Convexity in Inverse Coefficient Problems

Organizers: Henrik Garde and Bastian Harrach

MS11	**Wednesday, 16:30–16:55, SR 2.066**

A framework for tomography of nonlinear materials: the Monotonicity Principle

A. Corbo Esposito[1], **L. Faella**[1], **V. Mottola**[1], **G. Piscitelli**[2], **A. Tamburrino**[1,*]
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[2] *DiSEGIM, Università degli Studi di Napoli Parthenope, Italy.*
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The recent discovery of the Monotonicity Principle for problems involving nonlinear materials [1,2] makes it possible the development of tomographic methods in a completely new scenario. In this talk it is presented the complete framework [1-4] for dealing with tomography of nonlinear materials, with reference to the inverse obstacle problem where an unknown nonlinear anomaly is embedded in a linear background. Specifically, it will be presented the Monotonicity Principle for quasilinear elliptic PDEs, together with a real-time tomographic method [3] and the discussion of the theoretical limits and robustness against noise [4].

References

[1] A. Corbo Esposito, L. Faella, G. Piscitelli, R. Prakash, and A. Tamburrino, Monotonicity principle in tomography of nonlinear conducting materials, Inverse Problems, 37 (2021).

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[3] V. Mottola, A. Corbo Esposito, G. Piscitelli, and A. Tamburrino, Imaging of nonlinear materials via the monotonicity principle, Inverse Problems, 40 (2024), p. 035007.

[4] V. Mottola, A. Corbo Esposito, L. Faella, G. Piscitelli, R. Prakash and A. Tamburrino, The inverse obstacle problem for nonlinear inclusions. arXiv preprint arXiv:2408.08040 (2024).

MS11	Wednesday, 17:00–17:25, SR 2.066

The $p-$Laplace “Signature” for Quasilinear Inverse Problems

Antonio Corbo Esposito[1], **Luisa Faella**[1], **Vincenzo Mottola**[1], **Gianpaolo Piscitelli**[2,*], **Ravi Prakash**[3], **Antonello Tamburrino**[1]

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This presentation is focused on imaging problem encountered in the framework of Electrical Resistance Tomography involving two different materials, one or both of which are nonlinear. Tomography with nonlinear materials is in the early stages of developments, although breakthroughs are expected in the not-too-distant future.

We consider nonlinear constitutive relationships which, at a given point in the space, present a behaviour for large arguments that is described by monomials of order p and q.

The original contribution of this work makes is that the nonlinear problem can be approximated by a weighted $p-$Laplace problem. From the perspective of tomography, this is a significant result because it highlights the central role played by the $p-$Laplacian in inverse problems with nonlinear materials. Moreover, when $p = 2$, this provides a powerful bridge to bring all the imaging methods and algorithms developed for linear materials into the arena of problems with nonlinear materials.

MS11	Wednesday, 17:30–17:55, SR 2.066

Boundedness of the linearized forward map of EIT from square-integrable conductivity perturbations to Hilbert–Schmidt operators on square-integrable currents

Nuutti Hyvönen[1,*], **Joanna Bisch**[1], **Markus Hirvensalo**[1]

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This work considers the Fréchet derivative of the idealized forward map of two-dimensional electrical impedance tomography, i.e., the linear operator that maps a perturbation of the coefficient in the conductivity equation over a bounded two-dimensional domain to the linear approximation of the corresponding change in the Neumann-to-Dirichlet boundary map. Assuming the background conductivity coefficient is constant and the considered simply-connected domain has a $C^{1,\alpha}$ boundary, we show that the Fréchet derivative is bounded from the space of square-integrable conductivity perturbations to the space of Hilbert–Schmidt operators on the mean-free L^2 functions on the domain boundary [1]; see also [2] for closely related analysis. This result provides a Hilbert space framework for analyzing linearization-based one-step reconstruction algorithms of two-dimensional electrical impedance tomography.

References

[1] J. Bisch, M. Hirvensalo and N. Hyvönen, Continuity of the linearized forward map of electrical impedance tomography from square-integrable perturbations to Hilbert–Schmidt operators, *arXiv:2409.10671* (2024).

[2] H. Garde and N. Hyvönen, Linearised Calderón problem: Reconstruction and Lipschitz stability for infinite-dimensional spaces of unbounded perturbations *SIAM J. Math. Anal.* **56** (2024), pp. 3588–3604.

MS11 **Wednesday, 18:00–18:25, SR 2.066**

Monotonicity methods for solving the inverse problem of the time-harmonic elastic wave equation

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We solve the inverse problem of the time-harmonic elastic wave equation by means of monotonicity methods. In more detail, we start with the standard monotonicity method for shape reconstruction of inclusions in elastic bodies by using time-harmonic elastic waves as introduced in [1]. In order to accelerate the standard monotonicity method, we derive a linearized version. The linearized method provides an efficient version of the method, drastically reducing computation time (see [2]). In addition, we compare these methods with each other and present several numerical test examples including simulations based on real data from a lab experiment.

References

[1] S. Eberle-Blick and V. Pohjola, The monotonicity method for inclusion detection and the time harmonic elastic wave equation, *Inverse Problems*, **40** (2024), 045018.

[2] S. Eberle-Blick and V. Pohjola, The linearized monotonicity method for elastic waves and the separation of material parameters, *arXiv preprint* arXiv:2409.20339 (2024).

MS11 **Thursday, 11:30–11:55, SR -1.025**

One shot inverse scattering revisited

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As in [1,2], we consider the inverse acoustic obstacle scattering problem in the plane, given the far field for a *single* time-harmonic incident plane wave. In [2] Kusiak and Sylvester used this information to determine an approximation of the convex hull of the scatterers, whereas we have tried to distinguish multiple scatterers in [1], i.e., to determine a non-convex scattering support numerically. The present work is similar to [1] in spirit, with the essential difference being that we now only use the low-frequent (instead of the high-frequent) part of the far field, i.e., we restrict ourselves to angular frequencies up to the order of $\sqrt{\kappa r}$, where κ is the wave number and r is the diameter of the convex hull of the scatterers.

We employ the z-transform of the corresponding Fourier coefficients, and use a modification of the AAA algorithm from [3] to compute (complex) rational approximations of this function. It is shown that the polar angles of the poles of this rational function can be used to generate data for the Radon transform of the support of the scatterers. We present numerical examples to indicate the potential and limitations of this approach.

References

[1] M. Hanke, *SIAM J. Imaging Sci.* **5** (2012), pp. 465–482.

[2] S. Kusiak and J. Sylvester, *SIAM J. Math. Anal.* **36** (2005), pp. 1142–1158.

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MS11 **Thursday, 12:00–12:25, SR -1.025**

A remark on the local uniqueness by the monotonicity based method for the magnetic Schrödinger equation

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This study is based on the paper [3]. Inspired by the works of B. Harrach, V. Pohjola and M. Salo [2], which extended the previous result in [1], we study a local uniqueness by the monotonicity based method for the magnetic Schrödinger equation: $-D_A^2 u + qu = 0$ in Ω. Let $\Omega \subset \mathbb{R}^n (n = 2, 3)$ be a bounded domain with Lipschitz boundary $\partial\Omega$. Here, for $A = (A_1, A_2, \ldots, A_n)$, let $D_A^2 u := \sum_{j=1}^n D_{A,j}(D_{A,j} u)$, $\quad D_{A,j} := \frac{1}{i}\partial_j + A_j$. Throughout this paper, we assume that $A \in C^1(\overline{\Omega}; \mathbb{R}^n), q \in L^\infty(\Omega; \mathbb{R})$. In [3], we also give monotonicity tests to detect an unknown obstacle D for the magnetic Schrödinger equation. The unknown obstacle is assumed $D \subset \Omega$ with $\overline{D} \subset \Omega$ and connected complement $\Omega \setminus \overline{D}$.

References

[1] B. Harrach and M. Ullrich, *Local uniqueness for an inverse boundary value problem with partial data*, Proc. Amer. Math. Soc., **145** (2017), 1087-1095.

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[3] R. Yamashita, *A remark on the local uniqueness by the monotonicity based method for the magnetic Schrödinger equation,* Commun. Pure Appl. Anal. (2023), doi:10.3934/cpaa.2023077

MS11 **Thursday, 12:30–12:55, SR -1.025**

Monotonicity-based inversion formulae

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In this talk, we introduce some monotonicity methods combined with localized potentials, which are useful in the study of some inverse problems.

MS11 **Thursday, 15:00–15:25, SR 0.014**

Inverse medium scattering for a nonlinear Helmholtz equation

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The linear Helmholtz equation is used to model the propagation of sound waves or electromagnetic waves of small amplitude in inhomogeneous isotropic media in the time-harmonic regime. However, if the amplitudes are large then intensity-dependent material laws are required and nonlinear Helmholtz equations are more appropriate. A prominent example are Kerr-type nonlinear media. In this talk we discuss an inverse medium scattering problem for a class of nonlinear Helmholtz equations

$$\Delta u + k^2 u \,=\, -k^2 q(x,|u|)u\,, \qquad x \in \mathbb{R}^d\,,\ d = 2,3\,,$$

that covers generalized Kerr-type nonlinear media of the form

$$q(x,|z|) \,=\, q_0(x) + \sum_{l=1}^{L} q_l(x)|z|^{\alpha_l}\,, \qquad x \in \mathbb{R}^d\,,\ z \in \mathbb{C}\,,$$

where $q_0, \dots, q_L \in L^\infty(\mathbb{R}^d)$ with support in some bounded open set $D \subset \mathbb{R}^d$, the lowest order term satisfies $\operatorname{essinf} q_0 > -1$ in $\mathbb{R}^d$, and the exponents fulfill $0 < \alpha_1 < \cdots < \alpha_L < \infty$.

Assuming the knowledge of a nonlinear far field operator, which maps Herglotz incident waves to the far field patterns of the corresponding unique small solutions of the nonlinear scattering problem, we show that the nonlinear index of refraction is uniquely determined.

MS11 **Thursday, 15:30–15:55, SR 0.014**

A Monotonicity-Based Iterative Approach to Shape Reconstruction for the Helmholtz Equation

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We study an inverse boundary value problem on determining unknown coefficients for an inhomogeneous Helmholtz equation in a bounded region. We propose a monotonicity-based approach to identify the shape of unknown objects. This method involves minimizing a linearized residual function subject to a specific linear constraint within a monotonicity-based regularization. Using the monotonicity relation between the coefficients and the Neumann-to-Dirichlet operator, the global convergence of the minimization problem is established to reconstruct the shape of the unknown coefficients. Rigorous theoretical justifications and extensive numerical experiments are provided to verify the validity and feasibility of the proposed method.

Minisymposium 12

Computational Boundary Integral Equation Methods for Time-Dependent Waves

Organizers: Heiko Gimperlein and Jörg Nick

MS12	Monday, 11:45–12:10, SR 0.014

Space-time adaptive boundary elements for the weakly singular BIE related to the wave equation

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The transient wave equation with Dirichlet boundary conditions is here addressed by the Boundary Element Method (BEM). The differential problem is reformulated directly in time–domain by a single layer representation formula which induces a weakly singular Boundary Integral Equation (BIE). Then, the energy approach [1] suggests a well-established weak formulation whose discretization allows to recover the unknown density at the boundary, on the bounded time interval of analysis. Building on preliminary work in [2], we introduce a residual a posteriori error estimate which leads to an adaptive BEM with space-time mesh refinements. We will present several numerical results in $\mathbb{R}^2$, using both theoretical and heuristic error estimators, which will be compared.

References

[1] A. Aimi, M. Diligenti, C. Guardasoni, I. Mazzieri and S. Panizzi, An energy approach to space-time Galerkin BEM for wave propagation problems, *Int. J. Numer. Methods Engrg.* **80** (2009), pp. 1196–1240.

[2] H. Gimperlein, C. Ozdemir, D. Stark and E. P. Stephan, A residual a posteriori estimate for the time-domain boundary element method, *Numerische Mathematik* **146** (2020), pp. 239–280.

MS12 **Monday, 12:15–12:40, SR 0.014**

Time-Domain Combined Field Integral Equations for Scattering by Open Screens

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The time domain electric field integral equation is a versatile modelling tools that can solve the scattering problem by both closed and open perfect conductors. The number of iterations required to solve for the unknowns at subsequent time steps can be lowered by efficient Calderón preconditioning. Recently this technique has been extended to multi-screens [1,2]. Unfortunately the use of multi-screen methods results in kernels of the system matrix, which in conjunction with round-off and quadrature errors can be the source of late-time instabilities. In this talk we discuss a time-domain combined field integral equation for open screens [3] that does not suffer from this problem. It is demonstrated that introduction of the magnetic field integral equation term does not affect the solution but removes undesired kernels in the system matrix.

References

[1] X. Claeys and R. Hiptmair, Integral Equations for Electromagnetic Scattering at Multi-Screens, in *Integr. Equ. Oper. Theory*, **84:1** (2016)

[2] P. Olyslager, K. Cools, and H. Rogier, A Combined Field Integral Equation for Scattering by Open Surfaces, in *IEEE AP-S/URSI International Symposium*, (2024),

[3] K. Cools and C. Urzúa-Torres, Preconditioners for Multi-Screen Scattering, in *International Conference on Electromagnetics in Advanced Applications*, (2022), pp. 172–173.

MS12 **Monday, 12:45–13:10, SR 0.014**

Space-time BEM for the Wave Equation: What do we know so far?

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We present a novel framework for boundary integral equations for the wave equation. Unlike previous attempts, our mathematical formulation allows us to show that the associated boundary integral operators are continuous and satisfy inf-sup conditions in trace spaces of the same regularity, which are closely related to standard energy spaces. This property is crucial from a numerical point of view, as it establishes the foundations to derive sharper error estimates and paves the way to develop efficient adaptive space-time boundary element methods and stable space-time FEM/BEM coupling.

Moreover, the fact that the operators verify inf-sup conditions but not coercivity explains the unstabilities that practitioners have observed in the past. It also tells us that stability of space-time BEM for the wave equation is guaranteed if we choose boundary element spaces such that the discrete inf-sup condition is uniformly satisfied. In this talk, I will give an overview of the new framework and summarize what we have learnt so far.

References

[1] O. Steinbach, and C. Urzúa-Torres, A New Approach to Space-Time Boundary Integral Equations for the Wave Equation, *SIAM J. on Math. Analysis*, **54(2)** (2022), pp. 1370–1392.

MS12 **Monday, 15:00–15:25, SR 0.014**

Time-domain boundary integral equations and asymptotic models for scattering by small particles

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We consider a problem of time-domain acoustic wave scattering by multiple small particles of arbitrary Lipschitz-regular shapes, whose size ε tends to 0. In this talk we propose an asymptotic Foldy-Lax model to treat this problem.

In [1] we have suggested an approach to construct such a model as a Galerkin semi-discretization in space of a well-chosen time-domain boundary integral equation (TDBIE). The convergence is ensured by taking $\varepsilon \to 0$, rather than increasing the number of the basis functions. In this talk we will show how the ideas of [1], originally aimed at construction of a second-order model for circles, can be developed to handle obstacles of arbitrary shapes and to achieve high-order convergence. We will present error analysis and numerical experiments. Time permits, we address the question of the interplay between the small parameter ε and the final time in the error estimates.

References

[1] M. Kachanovska, A new class of uniformly stable time-domain Foldy-Lax models for scattering by small particles. Acoustic sound-soft scattering by circles, Multiscale Model. Simul. **22** (2024), no. 1, 1–38.

[2] M. Sini, H. B. Wang and Q. Yao, Analysis of the acoustic waves reflected by a cluster of small holes in the time-domain and the equivalent mass density, Multiscale Model. Simul. **19** (2021), no. 2, 1083–1114

MS12 **Monday, 15:30–15:55, SR 0.014**

Data Sparse Representation of a Convolution Quadrature based Time-Domain Boundary Element Method

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The time domain Boundary Element Method (BEM) for the homogeneous wave equation with vanishing initial conditions is considered. The generalized convolution quadrature method (gCQ) developed by Lopez-Fernandez and Sauter is used for the temporal discretisation. The spatial discretisation is done classically using low order shape functions. A collocation approach is applied on a Dirichlet problem solved with a single and double layer approach.

Essentially, the gCQ requires to establish boundary element matrices of the corresponding elliptic problem in Laplace domain at several complex frequencies. Consequently, an array of system matrices is obtained. This array of system matrices can be interpreted as a three-dimensional array of data which should be approximated by a data-sparse representation. The multivariate Adaptive Cross Approximation (3D-ACA) proposed by Bebendorf et al. can be applied to get a data sparse representation of these three-dimensional data arrays. Within the data slices corresponding to the BEM calculations at each frequency either the standard $\mathcal{H}$-matrices approach with ACA [1] or a fast multipole (FMM) approach can be used.

References

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MS12 **Monday, 16:30–16:55, SR 0.014**

The temporal domain derivative in inverse obstacle scattering

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We consider a time-dependent acoustic scattering problem, in which an incident wave impinges on a bounded, sound-soft obstacle. The resulting scattered wave as well as the incident wave both satisfy the time-dependent wave equation. In our inverse problem we are interested in reconstructing the obstacle from temporal measurements of the scattered wave at some observation points. In the context of time-harmonic wave propagation, as described, e.g., by the Helmholtz equation, an important tool for such reconstructions is the domain derivative, which can be understood as the linearization of the operator that maps the scattering object to the scattered wave away from the obstacle.

In this talk, we establish the temporal domain derivative for the acoustic wave equation in presence of a sound-soft scatterer and apply it to our inverse problem. We show that the temporal domain derivative is a solution to the wave equation away from the scatterer. The boundary condition on the obstacle as well as regularity requirements of the temporal domain derivative are deduced from studying the Helmholtz equation, which is the Laplace domain counterpart of the wave equation. We use the temporal domain derivative in a Gauß–Newton scheme in order to reconstruct the scattering object. Numerical examples highlight the feasibility of our approach.

MS12 **Monday, 17:00–17:25, SR 0.014**

Fast Boundary Element Methods for acoustic half space problems with impedance boundary condition in d spatial dimensions

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In our talk, we introduce a representation of the acoustic half space Green's function with impedance boundary conditions in d space dimensions which avoids oscillatory Fourier integrals. This new representation allows for fast approximation methods such as hierarchical matrices and efficient numerical quadrature.

This talk comprises joint work with Chuhe Lin, University of Zurich, and Markus Melenk, TU Vienna.

MS12 **Monday, 17:30–17:55, SR 0.014**

Fast Hybrid Frequency-time methods in wave scattering

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Hybrid frequency-time methods rely on temporal integral transforms such as the Fourier transform to synthesize transient solutions from frequency-domain problems. A method was recently proposed which efficiently constructs solutions to scattering problems with incident fields of arbitrary duration, with optimal asymptotic costs. Such fast hybrid methods proceed by windowing-and-recentering in time, solution in the frequency domain, and inversion using high-frequency numerical quadrature techniques so that ultimately only a fixed ($\mathcal{O}(1)$) set of frequency domain solutions enable high-order solution at arbitrarily large times with no dispersion error. This talk presents some recent developments including connections with recently-introduced decay theory and bootstrap domain-of-dependence bounds, 2D acceleration methods using matched long-time asymptotics, expansion of arbitrary smooth incident wavefields thereby enabling solution of the fully generic problem, and applications to dispersive media.

References

[1] T. G. Anderson, M. Lyon, O. Bruno *SIAM Journal on Scientific Computing* **42** (2020), pp. A1348-A1379.

Minisymposium 13

Wave Propagation in Random Multi-Scale Media and Applications

Organizers: Laure Giovangigli and Mathias Schäffner

MS13	Wednesday, 16:30–16:55, SR 3.069

Enhanced wave transmission in random media with mirror symmetry

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We present an analysis of enhanced wave transmission through random media with mirror symmetry about a reflecting barrier. The mathematical model is the acoustic wave equation, and we consider two setups, where the wave propagation is along a preferred direction: in a randomly layered medium and in a randomly perturbed waveguide. We use the asymptotic stochastic theory of wave propagation in random media to characterize the statistical moments of the frequency-dependent random transmission and reflection coefficients, which are scalar-valued in layered media and matrix-valued in waveguides. With these moments, we can quantify explicitly the enhancement of the net mean transmitted intensity, induced by wave interference near the barrier.

References

[1] L. Borcea and J. Garnier, Enhanced wave transmission in random media with mirror symmetry, *Proc. R. Soc. A* **480** (2024), 20240073.

MS13 **Wednesday, 17:00–17:25, SR 3.069**

Generalized Snell laws for rough interfaces

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In this talk, we revisit the problem of reflection and transmission of waves through a rapidly oscillating rough interface with general mixing properties. Under the paraxial (parabolic) scaling, the specular and speckle (diffusive) components of the reflected and transmitted waves can be precisely characterized. The scenario of a critically scaled interface will be presented, where the amplitudes of the interface fluctuations and the central wavelength are of the same order. When the correlation length of the interface fluctuations is smaller than the beam width, homogenization phenomena occur, and we will illustrate how the rough interface can be approximated as an effective flat interface, leading to deterministic specular cones. However, this scenario also results in the formation of broader cones (speckle cones) containing speckle patterns. We will present two-point correlation functions for these speckle patterns and a central limit type theorem, demonstrating that these patterns can be modeled as Gaussian random fields. This framework allows us to identify generalized Snell's laws for refraction and transmission, influenced by an effective scattering operator at the interface.

References

[1] C. Gomez and K. Sølna, Reflection and transmission problems for high-frequency waves at a randomly perturbed interface: generalized Snell's laws, *https://hal.science/hal-04317227*.

MS13 **Wednesday, 17:30–17:55, SR 3.069**

Validity of radiative transfer approximations in bounded random media

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This paper reports a series of numerical experiments comparing solutions of the 3D acoustic wave equation in a heterogeneous medium and of the radiative transfer equations. Parameters of the two equations are chosen such that the radiative transfer solution is expected to provide an accurate approximation of the energy of the wave in the weak scattering regime. The comparisons indicate that the radiative transfer provides accurate approximations even quite far from that regime. A particular attention is devoted to analyzing the results close to boundaries [1,2], where the accuracy of the radiative transfer equation has not been evaluated before.

References

[1] A. Messaoudi, R. Cottereau and C. Gomez, Boundary effects in radiative transfer of acoustic waves in a randomly-fluctuating half space, *Multiscale Modeling and Simulation: a SIAM Interdisciplinary Journal* **21** (2023), pp. 1299–1321. [doi] [arXiv]

[2] A. Messaoudi, C. Gomez and R. Cottereau, Weak localization in radiative transfer of acoustic waves in a randomly-fluctuating slab, Accepted for publication in *Wave Motion* **130** (2024), pp. 1–24, 2024. [doi] [arXiv]

MS13 **Thursday, 11:30–11:55, SR 3.068**

The time horizon for stochastic homogenization of the one-dimensional wave equation

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The wave equation with stochastic rapidly oscillating coefficients can be classically homogenized on bounded time intervals; solutions converge in the homogenization limit to solutions of a wave equation with constant coefficients. This is no longer true on large time scales: Even in the periodic case with periodicity ε, classical homogenization fails for times of the order ε^{-2}. We consider the one-dimensional wave equation with random rapidly oscillating coefficients on scale ε and are interested in the critical time scale $\varepsilon^{-\beta}$ from where on classical homogenization fails. In the general setting, we derive upper and lower bounds for β in terms of the growth rate of correctors. In the specific setting of i.i.d. coefficients with matched impedance, we show that the critical time scale is ε^{-1}.

References

[1] M. Schäffner and B. Schweizer. The time horizon for stochastic homogenization of the one-dimensional wave equation. *Asymptotic Analysis*, 2024.

[2] T. Dohnal, A. Lamacz and B. Schweizer. Bloch-wave homogenization on large time scales and dispersive effective wave equations. *Multiscale Model Simul*, 12(2):488–513, 2014.

MS13 **Thursday, 12:00–12:25, SR 3.068**

Long-time homogenization of the wave equation revisited

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Consider the linear wave equation with spatially oscillating coefficients in the setting of homogenization. At large times, the wave interacts in a nontrivial way with the heterogeneities, leading to effective dispersive effects. We present a new ansatz for the long-time two-scale expansion inspired by spectral analysis. Based on this spectral ansatz, we are able to extend and refine previous results, covering both the periodic and stationary random setting.

MS13 Thursday, 12:30–12:55, SR 3.068

Domain decomposition methods for random multi-scale Helmholtz problems arising in ultrasound imaging

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The development of new quantitative ultrasound imaging algorithms, which aim to reconstruct a map of the local ultrasound speed in the medium, requires a validation process that can be achieved through numerical simulation. With this application in mind, we consider the scattering of plane waves by a tissue-mimicking medium where up to a hundred unresolved scatterers per wavelength are randomly distributed throughout the medium. The domains (∼ about a hundred wavelengths in size) require billion degrees of freedom in a simulation, which corresponds to the state of the art in terms of direct numerical simulation capacity.

This work seeks to investigate the efficiency and scalability of two-levels domain decomposition techniques proposed in [2] to accurately solve the full scale model. The primary objective is to validate quantitative stochastic homogenization results obtained in [1], particularly the asymptotic expansions of the scattered field with respect to the size of the scatterers.

References

[1] J. Garnier, L. Giovangigli, Q. Goepfert, and P. Millien, Scattered wavefield in the stochastic homogenization regime, arXiv:2309.07777, 2023.

[2] F. Nataf and E. Parolin, Coarse spaces for non-symmetric two-level preconditioners based on local generalized eigenproblems, arXiv:2404.02758, 2024.

MS13 Thursday, 16:30–16:55, SR 3.068

Correlation-informed ordered dictionary learning for imaging in scattering media

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We propose an approach for imaging in scattering media when large and diverse data sets are available. Our approach has two steps. Using a dictionary learning algorithm, the first step estimates the Green's function vectors as unordered columns of a sensing matrix. The array data comes from many sparse sets of sources whose location and strength are not known. In the second step the columns of the estimated sensing matrix are ordered using Multi-Dimensional Scaling with connectivity information derived from cross correlations. Given these estimates, excellent imaging results are achieved. In strongly scattering media imaging resolution better than that in a homogeneous medium is achieved. This is known as super-resolution and it occurs because the scattering medium effectively enhances the physical imaging aperture.

References

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MS13 Thursday, 17:00–17:25, SR 3.068

Scattering by obstacles with periodic material properties, boundary correctors and inversion.

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We study the homogenization of a transmission problem arising in the scattering theory for bounded inhomogeneities with periodic coefficients in the Helmholtz equation, and we describe the boundary corrections. The coefficients are assumed to be periodic functions of the fast variable, specified over the unit cell with characteristic size ϵ. When the periodicity is in the squared index of refraction, we obtain improved convergence results that assume lower regularity than when the periodicity is in the second-order operator. In this case we describe the asymptotic behavior of boundary correctors for general domains at all orders. In particular we show that, in contrast to Dirichlet problems, the $O(\epsilon)$ boundary corrector is nontrivial and can be observed in the far field. We further demonstrate the latter far-field effect is larger than that of the "bulk" corrector – the so-called periodic drift, which is found to emerge only at $O(\epsilon^2)$. We also show how this expansion can be used to invert for the periodic structure. We illustrate the analysis by examples in one and two spatial dimensions.

MS13 Thursday, 17:30–17:55, SR 3.068

Scattering of waves from a densely and randomly arrayed set of very small and soft spherical objects

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We address the multiple scattering problem where randomly and densely distributed scatterers are very small, soft, and embedded in the wavefield, considering the transmission of waves into the scatterers. The characterization of these scatterers arises from fluctuations in mass density relative to the background wavefield structure. We investigate the asymptotic behavior of the wavefield as the spatial scale of the scatterers approaches zero, while keeping their total mass invariant. We construct the interaction equation for this problem using spherical harmonics expansions and ensuring the continuity of the wavefield across the boundaries of the scatterers. Notably, we do not directly apply the concept of the external field [1] to the interaction equation. Our discussions focus on how the present formulation aligns with the Foldy-Lax self-consistent method [2]. Additionally, we examine the causality of the wavefield derived from the Foldy algebraic equation. Finally, we present the results of several numerical computations to validate the present formulation by comparing them with those obtained from the boundary integral equation method.

References

[1] Foldy, L. L., The multiple scattering of waves. I. General theory of isotropic scattering by randomly distributed scatterers, *Physical Review*, **67**, pp. 107-119, 1945

[2] Martin, P. A., Multiple Scattering: Interaction of Time-Harmonic Waves with N Obstacles, Cambridge University Press, 2006.

Minisymposium 14

Analysis and Numerics for PDEs with Nonstandard and Mixed Boundary/Interface Conditions

Organizers: Martin Halla

MS14	Tuesday, 15:00–15:25, SR 2.067

Traces for Hilbert Complexes

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We study a new notion of trace operators and trace spaces for abstract Hilbert complexes. We introduce trace spaces as quotient spaces/annihilators. We characterize the kernels and images of the related trace operators and discuss duality relationships between trace spaces. We elaborate that many properties of the classical boundary traces associated with the Euclidean de Rham complex on bounded Lipschitz domains are rooted in the general structure of Hilbert complexes. We arrive at abstract trace Hilbert complexes that can be formulated using quotient spaces/annihilators. We show that, if a Hilbert complex admits stable "regular decompositions" with compact lifting operators, then the associated trace Hilbert complex is Fredholm. Incarnations of abstract concepts and results in the concrete case of the de Rham complex in three-dimensional Euclidean space will be discussed throughout.

References

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MS14	Tuesday, 15:30–15:55, SR 2.067

New estimates for potential operators in vector calculus and exterior calculus

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We discuss Poincaré–Friedrichs inequalities in the context of vector calculus and exterior calculus. This includes the Poincaré inequality for the gradient operator and its generalizations for the curl operator and the exterior derivative. Estimating the optimal constants in these inequalities reduces to obtaining estimates for the associated potential operators. We present several special cases and obtain upper bounds for convex Lipschitz domains using results by Guerini and Savo, along with new estimates for the regularized Poincaré and Bogovskii operators.

More generally, we examine Poincaré–Friedrichs constants over local finite element patches within triangulated domains, using the notion of shellability from the theory of polytopal complexes. Finally, we extend these results to general triangulated domains, deriving reliable and computable bounds for the Poincaré–Friedrichs constants of differential operators. A diagram chase within a Čech–de Rham complex reduces this to a merely finite-dimensional problem that is easily assembled from the geometric setting. Numerical experiments support the theoretical results. Part of this work is joint with Théophile Chaumont-Frelet and Martin Vohralík.

MS14	Tuesday, 16:30–16:55, SR 2.067

Numerics for the wave equation with nontrivial boundary conditions and nonlocal material laws

Selina Burkhard[1,*], **Marlis Hochbruck**[1]

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In this talk we consider the viscoacoustic wave equation with a kinetic boundary condition

$$\partial_t^2 u(t) - c^2 \Delta u(t) + \int_{-\infty}^{t} b(t-\theta)\Delta u(\theta)\,\mathrm{d}\theta = f(t), \tag{1a}$$

$$\partial_t^2 u(t) - c^2 \Delta_\Gamma u(t) + c^2 n\cdot\nabla u + \int_{-\infty}^{t} b(t-\theta)\Delta_\Gamma u(\theta)\,\mathrm{d}\theta + \int_{-\infty}^{t} b(t-\theta) n\cdot\nabla u(\theta)\,\mathrm{d}\theta = f_\Gamma(t) \tag{1b}$$

on a bounded domain Ω with smooth boundary Γ, equipped with suitable initial conditions. Here, n is the outer unit normal vector and Δ_Γ denotes the Laplace-Beltrami operator. We assume that the nonlinearity f is locally Lipschitz continuous.

We are interested in convolution kernels b being a linear combination of exponential kernels, which is a common model in geophysics. This special type of convolution kernel allows us to rewrite (1) as a first-order system of coupled PDEs by introducing auxiliary variables.

For the numerical approximation, we use isoparametric finite elements and propose an IMEX scheme for the time discretization. It treats the stiff linear part implicitly and the non-stiff nonlinear part explicitly. In the talk we will sketch the main ideas of the error analysis.

MS14 Tuesday, 17:00–17:25, SR 2.067

Frequency vs. time domain boundary conditions for viscothermal acoustics

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Effective frequency-domain boundary conditions that account for viscothermal losses in acoustics have recently gained in popularity due to their accuracy and computational efficiency. Emerging from a boundary-layer analysis of the linearized compressible Navier–Stokes equations, these can be classified as so-called Wentzell boundary conditions. Essentially, these conditions constitute an absorption–diffusion problem on solid boundaries, coupled through the flux at the boundary to the Helmholtz equation for the acoustic pressure in the interior. Generalizing this frequency-domain boundary-value problem to time domain, we obtain boundary conditions involving half-order fractional integral and differential operators for the viscous and thermal losses respectively. The inclusion of only the thermal part as a boundary condition to the isentropic wave equation yields a time-domain passive system of equations. However, when viscous losses are included, time passivity does not generally hold for the resulting initial–boundary-value problem, and a Briggs–Bears-type analysis of normal modes demonstrates unbounded exponential growth in time, indicating ill-posedness. Neglecting the viscous losses, the results of a finite-difference time-domain simulation of lossy sound propagation in a duct yield excellent agreement with corresponding frequency domain calculations. However, when viscous losses are included, these simulations experience an exponential instability in line with the Briggs–Bears analysis.

MS14 Tuesday, 17:30–17:55, SR 2.067

Generalized Impedance Boundary Conditions with sign-changing/vanishing impedance functions

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This talk is devoted to a 2D Laplace type problem with a generalized impedance boundary condition of the form $\partial_\nu u = -\partial_\tau(g(\tau)\partial_\tau u)$, where τ is the curvilinear abscissa of the boundary while ν is the normal coordinate. Such kind of boundary conditions appears for example as a result of an asymptotic analysis of problems involving small perturbations of the boundary or thin layers at the boundary [1]. Our talk focuses on cases when the function g vanishes or changes its sign. The main result is that the well-posedness in the Fredholm sense of the corresponding problems depends on how abruptly the function g vanishes or changes its sign. The obtained theoretical results are illustrated with the help of simple numerical experiments.

References

[1] B. Aslanyürek, H. Haddar and H. Şahintürk, Generalized impedance boundary conditions for thin dielectric coatings with variable thickness, *Wave Motion* **48** (2011), pp. 681–700.

[2] L. Bourgeois and L. Chesnel, Generalized Impedance Boundary Conditions with Vanishing or Sign-Changing Impedance, *SIAM Journal on Mathematical Analysis* **56** (2024), pp. 4223–4251.

MS14 **Wednesday, 11:30–11:55, SR 1.067**

Spectrum of the Maxwell Equations for a Flat Interface between Dispersive Media

Malcolm Brown[1], Tomas Dohnal[2,*], Michael Plum[3], Karl-Michael Schmidt[1], Ian Wood[4]

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The study of time harmonic electromagnetic waves at the interface of dispersive (i.e. frequency dependent) media leads to the non-self-adjoint operator pencil problem

$$\nabla \times \nabla \times E - \omega^2 \mu_0 \epsilon(x_1, \omega) E = 0, \quad \nabla \cdot (\epsilon E) = 0,$$

where $\epsilon(x_1, \omega) := \epsilon_0(1 + \hat{\chi}^{(1)}(x_1, \omega)) \in \mathbb{C}$ is the dielectric function and $\hat{\chi}^{(1)}$ is the electric susceptibility. A classical application is to surface plasmon polaritons at the interface of a metal and a dielectric. We assume that the interface is located at $x_1 = 0$, i.e. $\epsilon(\cdot, \omega)$ jumps at $x_1 = 0$ and the media in the two half spaces are either homogeneous or depend only on x_1. The dependence of ϵ on the spectral parameter ω (frequency) is generally nonlinear and we make no assumptions on its form.

The whole spectrum of the pencil L, with $L(\omega)E := \nabla \times \nabla \times E - \omega^2 \mu_0 \epsilon(\cdot, \omega)E$, consists of eigenvalues and the essential spectrum, but the various standard types of essential spectra do not coincide in all cases. The main tool for determining the essential spectra are Weyl sequences. The functional setting is such that the operator domain is not a subset of the range which brings about a difficulty in defining the discrete spectrum.

MS14 **Wednesday, 12:00–12:25, SR 1.067**

Dissipative boundary conditions, discrete spectra, and optimization of eigenvalues in bounded domains

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It is planned to start from the question how to formulate Leontovich or generalized impedance boundary conditions in such a way that they rigorously define an m-dissipative Maxwell or acoustic operator [1,3]. Then the discreteness of spectra will be discussed. These results are applied to structural optimization of eigenvalues associated with dissipative wave equations [2]. This can be seen as a multidimensional version for the problem of the decay rate minimization for 1-D resonances [4]. The talk is based primarily on two joint papers with Matthias Eller [1,2] and the recent preprint [3].

References

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[2] M. Eller and I.M. Karabash, Homogenization and nonselfadjoint spectral optimization for dissipative Maxwell eigenproblems. *Preprint arXiv:2401.01049* (2024), 31 p.

[3] I.M. Karabash, Random acoustic boundary conditions and Weyl's law for Laplace-Beltrami operators on non-smooth boundaries. *Preprint arXiv:2410.15150* (2024), 35 p.

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MS14	Wednesday, 12:30–12:55, SR 1.067

A posteriori error bounds for the finite element discretization of wave propagation problems in unbounded domains

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In this talk, I will consider Helmholtz problems set in unbounded domains Ω where the radiation condition is taken into account by (non-truncated) Perfectly Matched Layers (PML). The approximation procedure for such problems is typically divided into two steps. First (i), a modeling error is introduced by truncating the PML at some finite depth L. Then (ii), the remaining bounded domain Ω^L is meshed with elements of size h and a finite element space $V_h \subset H^1_0(\Omega^L)$ of degree p. Standard a posteriori error estimators for this problem only take into account the error incurred in step (ii), but disregard the modeling error introduced in step (i).

I will discuss an alternative viewpoint where Ω is discretized by an infinite mesh of size h, on which a finite-dimensional finite element space of degree p is constructed. The construction of the finite element space $V_h^L \subset H^1_0(\Omega^L)$ implicitly involves a truncation procedure, so that the situation is formally identical to the standard setting. However, the key insight of this interpretation is that one can construct an error estimator that accounts for all the sources of error induced by the discretization. This estimator can in particular steer adaptive mesh refinement algorithms that automatically adjust the truncation of the PML.

MS14	Wednesday, 15:00–15:25, SR 1.067

Learned infinite elements for the vector-valued time-harmonic Galbrun's equation

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Galbrun's equation is a vector-valued PDE describing the propagation of time-harmonic acoustic waves in the presence of a steady background flow and is of central interest for the field of helioseismology – the study of the sun through its oscillations. For realistic simulations, it is essential to impose transparent boundary conditions. However, due to the highly variable physical parameters within the Sun, standard methods like Perfectly Matched Layers (PML) or classical learned infinite elements are ineffective. To address this, learned infinite elements (LIEs) were introduced [1] and have proven to be successful for scalar-valued problems. In this talk, we demonstrate how LIE can be applied to the vector-valued Galbrun's equation by leveraging a scalar-valued exterior problem derived in [2].

References

[1] M. Halla, On the treatment of exterior domains for the time-harmonic equations of stellar oscillations, *SIAM J. Math. Anal.*, Vol. 54, Iss. 5, pp. 5268-5290 (2022).

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MS14 **Wednesday, 15:30–15:55, SR 1.067**

Target Signatures for Anisotropic Sheets in Electromagnetic Scattering

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Target signatures have been suggested as a technique for detecting changes in scatterers using single frequency time-harmonic scattering information. They typically arise as eigenvalues of some interior problem, but can be detected from far field data. In this talk I will discuss preliminary work on target signatures for ultra-thin screens in two dimensions governed by the Helmholtz equation. I shall then move on to more recent work on target signatures for anisotropic screens in three dimensions, where the wave field is governed by the time-harmonic Maxwell's equations. These problems typically give rise to eigenvalue problems with non-standard boundary conditions, and in the three dimensional case anisotropy plays an important practical role.

Minisymposium 15

Novel Approaches to Inverse Problems in Wave Scattering and Propagation

Organizers: Isaac Harris and Jacob Rezac

MS15	**Tuesday, 15:00–15:25, SR 3.069**

Solving inverse problems involving shock waves by following particle trajectories

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We study inverse problems for nonlinear hyperbolic conservation laws. Given noisy observations of the entropy solution, we consider the problem of identifying the initial field or the flux function. It is well-known that solutions of hyperbolic conservation laws exhibit discontinuities which travel along the flow known as *shock waves.* This poses several challenges to both forward and inverse problems. In particular, due to shock waves, direct observations of the entropy solution are not "regular" enough to fit in the Bayesian framework in Stuart (2010). To get round this, we propose a new approach by studying the trajectories for hyperbolic conservation laws and exploring their existence, uniqueness and stability. This talk is based on [1,2].

References

[1] D.L. Duong, *Inverse problems for hyperbolic conservation laws: a Bayesian approach*, PhD Thesis, University of Sussex, 2021.

[2] M. Dashti and D.-L. Duong, Stability of particle trajectories of scalar conservation laws and applications in Bayesian inverse problems, *arXiv:2307.14536* (2023).

MS15 **Tuesday, 15:30–15:55, SR 3.069**

Inverse Schrödinger scattering via data-driven reduced order models

Andreas Tataris[1,*], **Tristan van Leeuwen**[2], **Alexander Mamonov**[3]

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In this talk, we present a numerical method for solving an inverse scattering problem of estimating the scattering potential in a Schrödinger equation from frequency domain measurements based on reduced order models (ROM). The ROM is a projection of Schrödinger operator onto a subspace spanned by its solution snapshots at certain wavenumbers. Provided the measurements are performed at these wavenumbers, the ROM can be constructed in a data-driven manner from the measurements on a surface surrounding the scatterers. Once the ROM is computed, the scattering potential can be estimated using non-linear optimization that minimizes the ROM misfit. Such an approach typically outperforms the conventional methods based on data misfit minimization. During the talk we will showcase two variants of ROM-based algorithms for inverse scattering and present some numerical results in 2D.

MS15 **Tuesday, 16:30–16:55, SR 3.069**

Inverse electromagnetic scattering problems for long tubular objects

Raphael Schurr[1,*], **Tilo Arens**[1], **Marvin Knöller**[1]

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We consider the scattering of time harmonic electromagnetic waves by long tubular objects, motivated by applications in which such objects are to be shape-optimized with respect to chirality properties. As a preliminary study, we here consider the inverse shape reconstruction problem for a scatterer from far field of the scattered electromagnetic wave. We characterize the domain derivative of the scattered field for the special class of tubular objects and implement a corresponding shape reconstruction algorithm. The underlying analysis is supported by the numerical results we will present. The implementation of the shape reconstruction algorithm directly feeds into the ultimate goal of realizing a shape optimization algorithm for such scatterers with respect to chiral properties.

References

[1] T. Arens, M. Knöller, and R. Schurr, Inverse electromagnetic scattering problems for long tubular objects, *CRC 1173 Preprint 2024/17.*

MS15 **Tuesday, 17:00–17:25, SR 3.069**

Reconstruction of acoustic medium properties and source in time-domain by injecting contrasting agents

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In this talk, we will discuss an inverse problem in the time-domain wave equation which amounts to simultaneously reconstruct the medium properties (such as mass density and bulk modulus) and source function. For the data, along with traditional measurement i.e. the Dirichlet/Neumann trace of wave-field on the boundary, we consider an auxiliary measurement in terms of the boundary observation of wave-field generated by the medium when it is injected by small-scaled contrast agents at different interior points. Under critical scaling of these contrast agents, we derive an asymptotic profile for the auxiliary wave-field which results in simultaneous reconstruction of material properties and source. Our analysis primarily uses the time-domain Lippmann-Schwinger equation and integral equation techniques.

This talk will be based on the works [1] and [2].

References

[1] S. Senapati and M. Sini, Minnaert frequency and simultaneous reconstruction of the density, bulk and source in the time-domain wave equation, *arXiv: 2311.08114*.

[2] S. Senapati, M. Sini and H. Wang, Recovering both the wave speed and the source function in a time-domain wave equation by injecting contrasting droplets, in *Discrete and Continuous Dynamical Systems. Series A,* (2024), volume 44, no. 5 , 1446-1474.

MS15 **Tuesday, 17:30–17:55, SR 3.069**

An efficient D-N alternating algorithm for solving an inverse problem for Helmholtz equation.

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[2] *Laboratoire de Mathematiques Jean Leray, Nantes Universite, Nantes, France*
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Data completion known as Cauchy problem is one most investigated inverse problems. In this work we consider a Cauchy problem associated with Helmholtz equation. Our concerned is the convergence of the well-known alternating iterative method [1]. Our main result is to restore the convergence for the classical iterative algorithm (KMF) when the wave numbers are considerable. This is achieved by, some simple modification for the Neumann condition on the under-specified boundary and replacement by relaxed Neumann ones. Moreover, for the small wave number k , when the convergence of KMF algorithm's [1] is ensured, our algorithm can be used as an acceleration of convergence. In this case, we present theoretical results of the convergence of this relaxed algorithm. Meanwhile it, we can deduce the convergence intervals related to the relaxation parameters in different situations. In contrast to the existing results, the proposed algorithm is simple to implement converges for all choice of wave number.

We approach our algorithm using finite element method to obtain an accurate numerical results, to affirm theoretical results and to prove it's effectiveness.

References

[1] V. A. Kozlov, V. G. Maz'ya and A. V. Fomin, An iterative method for solving the Cauchy problem for elliptic equations, *Comput. Math. Phys* **31** (1991), pp. 45–52.

MS15 **Wednesday, 11:30–11:55, SR 2.067**

Optimization strategies for mitigating challenges in radar qualitative inverse scattering

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The phase-encoded linear sampling method (PE-LSM) is a qualitative inverse scattering (QIS) technique for reconstructing the shape of an unknown target. Unlike most QIS techniques, it is formulated to allow for high-fidelity imaging from a single moving sensor. It is also capable of achieving higher resolution than traditional radar focusing [1]. The PE-LSM thus shows promise for single-sensor radar imaging applications, such as synthetic aperture radar.

Previous PE-LSM work has mostly used controlled and well-characterized imaging environments. In this study, we extend the development to account for signal corruptions that may occur in more challenging field scenarios and which may mask or degrade the target response. These corruptions may originate from clutter responses from terrain surrounding the target, obscurations between the target and sensor such as foliage, and imperfectly compensated platform motion.

To mitigate these effects, we develop an optimization-based framework for estimating and compensating the unwanted signal modulations. We then evaluate performance by applying the techniques to a diverse collection of datasets.

References

[1] M. Burfeindt, H. Alqadah, and S. Ziegler, "Experimental phase-encoded linear sampling method imaging with a single transmitter and receiver," *IEEE Open J. Ant. Prop.*, vol. 5, no. 4, pages 942–957, 2024.

MS15 **Wednesday, 12:00–12:25, SR 2.067**

Application of the topological derivative to solve the inverse scattering problem from experimental data.

Ana Carpio[1], **Manuel Pena**[2,*], **María-Luisa Rapún**[2]
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The topological derivative (TD) is a very useful tool for the solution of inverse problems. In particular, when applied to problems modelled by the wave equation, it is able to correctly identify both sound-hard and sound-soft scatterers, as well as penetrable ones. It does it with a single iteration algorithm which, in the case of problem with an explicit formula for the fundamental solution, does not requires numerical solution of any PDE.

In this talk we will summarize the results obtained in [2,4] when testing indicators based on the TD, against two experimental databases [1,3] with electromagnetic radiation measurements.

References

[1] K. Belkebir and M. Saillard, Testing inversion algorithms against experimental data. *Inverse problems* **17** (2001), p. 1565.

[2] A. Carpio, M. Pena, and M-L. Rapún, Processing the 2D and 3D Fresnel experimental databases via topological derivative methods. *Inverse Problems* **37** (2021), p. 105012.

[3] J. M. Geffrin and P. Sabouroux, Continuing with the Fresnel database: experimental setup and improvements in 3D scattering measurements. *Inverse Problems* **25** (2009), p. 024001.

[4] M. Pena, S. Muñoz, and M-L. Rapún, Exploring the performance of the topological energy method for object and damage detection from noisy and poor databases. *Philosophical Transactions A* **382** (2024), p. 20230303.

MS15 **Wednesday, 12:30–12:55, SR 2.067**

Non-scattering wave numbers versus transmission eigenvalues

Andreas Kleefeld[1,*], **Lukas Pieronek**[2]
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An important question arising in inverse problems for wave scattering is whether for a given inhomogeneous bounded obstacle in two dimensions there is an incident wave that does not scatter. Closely connected to this question is the solution of the interior transmission problem. Therefore, let A be the discrete set of non-scattering wave numbers and B be the discrete set of transmission eigenvalues. It is well-known that $A = B \neq \emptyset$ holds for a disk and $B \supsetneq A = \emptyset$ holds for a square. The questions remains whether there is a bounded obstacle for which $A \subsetneq B$ with $A \neq \emptyset$. To address this question, the problem at hand is recasted as a constrained optimization problem using Fourier-Besselfunctions and then finally solved numerically. Some numerical results are presented and interesting observations are made both of which merit further investigation.

MS15 **Wednesday, 15:00–15:25, SR 2.067**

Applications of sparse optimization to inverse scattering and wireless communications

Jacob D Rezac[1,*]
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We discuss generalizations of a classical signal processing technique and apply them to two inverse problems: estimating the angle-of-arrival of a radio wave impinging on an array of antennas and estimating the location and shape of an inhomogeneity from scattered acoustic waves. We discuss this signal processing technique, the MUSIC algorithm (Multiple Signal Classification), as a method to relate unknowns-of-interest to the range of a matrix containing measurements of a relevant physical quantity in each of these applications. We build two new algorithms on this relationship, one which improves computational efficiency and one which heavily reduces measurement requirements. Each new algorithm is based on a technique from sparse optimization. We demonstrate these techniques on measured and simulated examples.

MS15 **Wednesday, 15:30–15:55, SR 2.067**

Extending Qualitative Methods to Biharmonic Scattering

Isaac Harris[1,*]

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This talk will discuss extending the direct sampling method (DSM) to inverse shape problems for Biharmonic Scattering. This method is a computationally simple and analytically rigorous way to define an imaging function to recover the scatterer. Here we will focus on the case of scattering by a clamped region by an incident plane wave. Using the far-field data, we will analyze the DSM and prove it's a stable reconstruction method. The techniques for analyzing the DSM are similar to the standard Helmholtz equation with a few interesting surprises we will discuss how to tackle. Numerical examples are given to show the applicability of the new imaging functionals.

Minisymposium 16

(In-)Stability, Dispersion and Weak Turbulence: From Fluid Flow to Hamiltonian Systems

Organizers: Zihui He, Lars Eric Hientzsch, and Tobias Schmid

MS16	Wednesday, 11:30–11:55, SR 0.014

On the effect of rotation or stratification on the stability of viscous 3d Couette flow

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As is well known, perturbations of Couette flow in the Navier-Stokes equations experience phase mixing, which stabilizes fluid motion. In the presence of suitable rotational forces or stratification, additionally dispersive internal gravity or inertial waves arise. These two mechanisms are of fundamentally different nature and relevant in complementary dynamical regimes. We will discuss how their combined effect leads to a quantitatively improved stability theory over the case of pure Couette flow.

This is based on joint work with M. Coti Zelati and A. Del Zotto.

MS16 **Wednesday, 12:00–12:25, SR 0.014**

On the nonlinear transverse asymptotic stability of line solitary waves for the three dimensional Euler-Poisson system.

Frédéric Rousset[1], **Changzhen Sun**[2,*]

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It is known from [1] that the one-dimensional Euler-Poisson system admits a family of solitary waves with exponential localization, which are also solutions to the higher-dimensional Euler-Poisson system. In this talk, we examine the stability of this family of line solitary waves for the three-dimensional Euler-Poisson system. Since the perturbation is allowed to depend on the transverse variable, this type of stability is typically referred to as transverse stability.

Our findings show that the line solitary waves are transversely stable, and the solution, starting near the line solitary waves, exists all the time and tends to some modulated waves in the L^∞_x norm as time approaches infinity.

References

[1] J. Bae and B. Kwon. Small amplitude limit of solitary waves for the Euler-Poisson system. J. Differential Equations, 266(6):3450–3478, 2019.

MS16 **Wednesday, 12:30–12:55, SR 0.014**

Two-dimensional Navier-Stokes equations with freely transported viscosity coefficient

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In this talk we consider the two-dimensional incompressible Navier-Stokes equations, where the viscosity coefficient may exhibit big jumps across some regular interfaces that are freely transported by the fluid's velocity field. We investigate the interplay between growth due to transport and decay due to dissipation effects, with a detailed analysis of tangential regularity. Assuming a scaling-invariant smallness assumption on the initial data, we achieve global-in-time regularity through a bootstrap argument. Other relevant inhomogeneous models are also discussed.

MS16 **Wednesday, 16:30–16:55, SR 3.068**

One-Dimensional Energy Cascade in a Quasi-Linear Dispersive Equation

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We study the transfer of energy to high frequencies in a quasi-linear Schrödinger equation with a sublinear dispersion relation on the one-dimensional torus, a toy model for gravity water waves. We construct initial data that exhibit finite-time Sobolev norm growth: starting with arbitrarily small norms in high-regularity spaces, these norms become arbitrarily large at later times.

Our analysis identifies an instability mechanism driving this energy cascade. Using para-differential normal forms, we derive an effective equation governed by a transport operator with non-constant coefficients. A positive commutator method inspired by Mourre's theory reveals how this operator induces the instability.

We believe that our work provides a foundational step toward investigating energy cascades in more complex fluid models, including gravity water waves.

References

[1] Maspero A., Murgante F.: *One dimensional energy cascades in a fractional quasilinear NLS.* https://doi.org/10.48550/arXiv.2408.01097

MS16 **Wednesday, 17:00–17:25, SR 3.068**

(In)stability of multi-soliton solutions in dispersive systems with periodic potential

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We present an abstract framework to establish the existence and spectral (in)stability of multiple front and pulse solutions, along with their periodic extensions, in semilinear evolution problems with spatially periodic heterogeneities. We adopt a spatial dynamics approach to construct these solutions near formal superpositions of well-separated primary fronts or pulses by characterizing invertibility through exponential dichotomies. With the aid of Evans functions we prove that the spectrum of the system's linearization about such a solution converges to the union of the spectra of the primary fronts or pulses as the distances between them tend to infinity. Our methods can be applied to the dispersive Gross-Pitaevskii and Lugiato-Lefever equations with periodic potentials, yielding the existence and spectral (in)stability of multi-soliton solutions near formal superpositions of black or bright NLS solitons.

MS16 **Wednesday, 17:30–17:55, SR 3.068**

On the transverse stability of smooth solitary waves in a two-dimensional Camassa–Holm equation

Anna Geyer[1,*], **Yue Liu**[2], **Dmitry Pelinovsky**[3]

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The Camassa-Holm equation is well-known for modeling the unidirectional propagation of waves in shallow water. In this talk, we consider a two-dimensional generalisation of the Camassa-Holm equation which is similar to the way the KP equation extends the famous KdV equation to two dimensions. We will study the transversal stability of its solitary traveling wave solutions, that is, stability with respect to perturbations that are transverse to the direction of propagation. To this end we study the spectrum of an operator which arises after linearisation around a perturbation of the solitary wave in suitably weighted spaces. We show that the double eigenvalue of the linearized equations related to the translational symmetry breaks under such transverse perturbations into a pair of asymptotically stable resonances and that the continuous spectrum is located in the left half-plane. Moreover, we prove that small-amplitude solitary waves are linearly stable with respect to transverse perturbations by performing careful resolvent estimates and making use of an asymptotic reduction of CH to KdV. The talk is based on joint work with Yue Liu and Dmitry Pelinovsky [1].

References

[1] A. Geyer, Y. Liu and D. Pelinovsky, On the transverse stability of smooth solitary waves in a two-dimensional Camassa-Holm equation, *Journal de Mathématiques Pures et Appliquées* **188** (2024), pp.1–25.

MS16 **Wednesday, 18:00–18:25, SR 3.068**

Asymptotic stability of the Boussinesq model solitary waves in the energy space.

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In this talk, I will present the stability problem for the solitary waves of some models from the Boussinesq family. We will give a brief overview of previous results on the stability problem. Then, we will present some recent developments on the stability of solitary waves, and by using an approach based on virial identities, inspired by the work of Kowalczyk, Martel, Muñoz, and Van den Bosch, we will prove that these solutions are asymptotically stable for any perturbation in the energy space. Part of this work was done jointly with Claudio Muñoz.

MS16 **Thursday, 11:30–11:55, SR 0.014**

Perturbation at blow up of self similar solution for mKdV

Raphaël Côte[1,*], **Simão Correia**[2], **Luis Vega**[3]
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The modified Korteweg-de Vries equation (mKdV) is an asymptotic model for fluid dynamics, and its self-similar solutions are connected to the formation of spirals and corners in a vortex patch.
In this talk, I will present some recent results regarding the description, stability and perturbation of the blowup dynamic of self-similar solutions of (mKdV).

References

[1] Simão Correia and Raphaël Côte, Sharp blow-up stability for self-similar solutions of the modified Korteweg-de Vries equation. *Preprint* arXiv:2402.16423

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[4] Simão Correia, Raphaël Côte et Luis Vega, Asymptotics in Fourier space of self-similar solutions to the modified Korteweg-de Vries equation. *J. Math. Pures Appl.* **137** (2020), 101–142.

MS16 **Thursday, 12:00–12:25, SR 0.014**

Zero-dispersion limit for the Benjamin-Ono equation

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We focus on the Benjamin-Ono equation on the line with a small dispersion parameter. The goal of this talk is to precisely describe the solution at all times when the dispersion parameter is small enough. This solution may exhibit locally rapid oscillations, which are a manifestation of a dispersive shock. The description involves the multivalued solution of the underlying Burgers equation, obtained by using the method of characteristics. This work is in collaboration with Elliot Blackstone, Patrick Gérard, Peter D Miller and Matthew D Mitchell.

MS16 **Thursday, 12:30–12:55, SR 0.014**

Global Well-Posedness and Soliton Resolution for the Half-Wave Maps Equation with Rational Data

Enno Lenzmann[1,*]

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In this talk, I discuss the energy-critical half-wave maps equation (HWM) which can be written as

$$\partial_t \mathbf{u} = \mathbf{u} \times |D|\mathbf{u}$$

where $\mathbf{u} : [0,T) \times \mathbb{R} \to \mathbb{S}^2$. It has been known for quite some time that (HWM) is completely integrable with a Lax pair structure. However, despite this striking feature, the question about global-in-time existence of solutions has been completely open so far – even for smooth and sufficiently small initital data. I will present recent results that show global well-posedness for rational initial data (with no size restriction) along with a soliton resolution result in the large-time limit. The proofs strongly exploit the Lax structure of (HWM) combined with an explicit flow formula. This is joint work with P. Gérard (Paris-Saclay).

Minisymposium 17

Analysis and Numerical Simulation of Wave Scattering by Periodic Structures

Organizers: Guanghui Hu and Ruming Zhang

MS17	Tuesday, 16:30–16:55, SR 0.014

Time harmonic Maxwell's equations in periodic waveguides

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We study time harmonic Maxwell's equations in a closed waveguide,

$$\operatorname{curl} E = i\omega\mu\, H + f_h\,, \qquad \operatorname{curl} H = -i\omega\varepsilon\, E + f_e\,,$$

in a domain $\Omega = \mathbb{R} \times S \subset \mathbb{R}^3$, where $S \subset \mathbb{R}^2$ is a bounded Lipschitz domain. Given are the coefficients $\mu = \mu(x)$ (permeability) and $\varepsilon = \varepsilon(x)$ (permittivity), they are both depending on the spatial position $x \in \mathbb{R}^3$ and are periodic in x_1. A functional analytic approach is used to formulate and to solve the radiation problem. We furthermore characterize the set of all bounded solutions to the homogeneous problem and treat the case of a compact perturbation of the medium.

References

[1] S. Fliss and P. Joly. Solutions of the time-harmonic wave equation in periodic waveguides: asymptotic behaviour and radiation condition. *Arch. Ration. Mech. Anal.*, 219(1):349–386, 2016.

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MS17 **Tuesday, 17:00–17:25, SR 0.014**

An efficient method for solving non-periodic scattering problems in locally perturbed periodic structures

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We consider scattering of non-periodic incident fields by locally perturbed periodic structures. Such scattering problems, modeled by the Helmholtz equation, are challenging due to the unboundedness of the domain and disruption of the periodicity. To overcome these difficulties, the local perturbation is removed via a coordinate transform at the cost of dealing with non-homogeneous coefficients. To truncate the problem away from the scatterer, we use the perfectly matched layer (PML) elaborated in [1]. Afterward, the Floquet-Bloch transform is applied to decompose the non-periodic problem posed in the periodic domain into a coupled family (indexed by the Floquet parameter) of periodic problems in a bounded cell. Due to the coupling, setting up and solving the resulting linear system directly is time and memory consuming [2]. We propose a novel reformulation of the linear system by the Schur complement and derive an efficient method to solve the system in parallel. Some numerical results illustrate the efficiency of the method and the convergence rate with respect to the PML parameter.

References

[1] S. N. Chandler-Wilde and P. Monk, The PML for rough surface scattering, *Appl. Numer. Math.* **59** (2009), pp. 2131–2154.

[2] R. Zhang, A High Order Numerical Method for Scattering from Locally Perturbed Periodic Surfaces, *SIAM J. Sci. Comput.* **40** (2018), pp. A2286–A2314.

MS17 **Tuesday, 17:30–17:55, SR 0.014**

Analysis of RCWA

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The scattering matrix algorithm (SMA), also known as rigorous coupled wave analysis (RCWA) or as Fourier modal method (FMM), is a popular numerical algorithm for the simulation of the diffraction of plane waves by grating structures (cf., e.g., [1]). Based on a slicing of the grating domain, it can be considered as a non-overlapping domain-decomposition method, where the final global discretized equation is resolved by an iteration over the subdomain equations. Using the RCWA or FMM, the solution over each of the subdomains is obtained by solving initial value problems for operator valued ordinary differential equations equivalent to the wave equation. In our talk we discuss the different convergence conditions for the TE and TM polarization in case of the two-dimensional Helmholtz equation (cf. [2,3,4]).

References

[1] M. Nevière and E. Popov, *Light propagation in periodic media*, Marcel Dekker, Inc., New York, Basel, 2003.

[2] B.J. Civiletti, A. Lakhtakia, and P.B. Monk, Analysis of the Rigorous Coupled Wave Approach for p-polarized light in gratings, *J. Comp. Appl. Math.* **386** (2021), 113235.

[3] G. Hu and A. Rathsfeld, *Radiation conditions for the Helmholtz equation in a half plane filled by inhomogeneous periodic material*, WIAS Preprint **2726**, Berlin 2020.

[4] A. Rathsfeld, *Convergence of the method of rigorous coupled-wave analysis for the diffraction by two-dimensional periodic surface structures*, WIAS Preprint **3081**, Berlin 2023.

MS17 | **Tuesday, 18:00–18:25, SR 0.014**

Radiation condition for the Helmholtz equation in an infinite periodically inhomogeneous medium

Guanghui Hu[1], **Andreas Rathsfeld**[2], **Jiayi Zhang**[1,*]

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In this talk, we propose a new radiation condition of the Helmholtz equation for an inhomogeneous medium which is periodic in the vertical direction in two dimensions. The classical Rayleigh-expansion radiation condition is valid only in a homogeneous medium and does not apply to our case. We utilize the Floquet theory to obtain a basis of upgoing and downgoing wave modes and define the radiation condition by expansions w.r.t. these modes. Furthermore, we prove the boundedness of the Dirichlet-to-Neumann map and decompose it into the sum of a coercive operator and a compact operator. Finally, we verify the Fredholm property of the variational formulation for time-harmonic scattering of plane waves from such an inhomogeneous medium occupying a half space. With the assumption of uniqueness at one frequency, we prove the well-posedness of the scattering problem all but a discrete set of frequencies.

MS17 | **Wednesday, 11:30–11:55, SR 3.068**

Extreme Bound States in the Continuum in Simple Periodic Structures

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A bound state in the continuum (BIC) corresponds to an eigenvalue embedded in the continuous spectrum, and is associated with the non-uniqueness of a related scattering problem [1]. In a periodic structure with a single periodic direction y and sandwiched between two homogeneous media, a BIC has a frequency ω_* and a Bloch wavenumber β_*, and it is surrounded by resonant modes with a real β and a complex frequency ω. A resonant mode satisfies an outgoing radiation condition, decays with time, and has a finite quality factor (Q factor) $Q = -0.5\mathrm{Re}(\omega)/\mathrm{Im}(\omega)$. A larger Q leads to stronger resonant wave phenomena, such as local field enhancement. Near a typical BIC, $Q \sim 1/(\beta - \beta_*)^2$, but for special BICs, $Q \sim 1/(\beta - \beta_*)^m$ for $m \geq 4$ [2,3]. In this talk, we reveal some extreme BICs in a very simple periodic structure, for which $Q \sim 1/(\beta - \beta_*)^{10}$.

References

[1] C. W. Hsu, B. Zhen, A. D. Stone, J. D. Joannopoulos, and M. Soljačić, "Bound states in the continuum," Nat. Rev. Mater. **1**, 16048 (2016).

[2] L. Yuan and Y. Y. Lu, "Bound states in the continuum on periodic structures surrounded by strong resonances," Physical Review A **97**(4), 043828 (2018).

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MS17 **Wednesday, 12:00–12:25, SR 3.068**

Spatiotemporal photonic crystals and their homogenization

Carsten Rockstuhl[1,*], **Puneet Garg**[1], **Aristeidis G. Lamprianidis**[1], **Sydur Rahman**[1], **Michael Plum**[1], **Barbara Verfürth**[2]
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Spatiotemporal photonic crystals are a new class of photonic media characterized by spatial periodicity, as known for ordinary photonic crystals. However, the material properties are also periodic in time. While spatial periodicity introduces energy gaps, temporal periodicity introduces momentum gaps. When both the spatial and temporal modulation period is much higher than the spatial and temporal oscillation period of an electromagnetic field that propagates through such a medium, homogenization is possible, and the field effectively experiences a stationary medium.

In this contribution, we develop a homogenization theory that assigns effective properties to the spatiotemporal photonic crystals in such a regime. Specifically, we explore a two-step procedure. There, initially homogenization is done in the temporal domain, and in the second step, a spatial homogenization is done. Eventually, we reach at local constitutive relations, but extensions to accommodate also nonlocal aspects in the homogenization procedure are discussed.

References

[1] P. Garg, A. G. Lamprianidis, S. Rahman, N. Stefanou, E. Almpanis, N. Papanikolaou, B. Verfürth, and C. Rockstuhl, Two-step homogenization of spatiotemporal metasurfaces using an eigenmode-based approach, *Optics Materials Express* **14** (2024), pp. 549–563

MS17 **Wednesday, 12:30–12:55, SR 3.068**

Chiral optical cavities using metasurfaces of highly electromagnetically chiral scatterers

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Chirality is the geometric concept of an object and its mirror image being non-superimposable. It is ubiquitous in nature and is found, for instance, in molecules that exist as two opposite-handed isomers, called enantiomers. The ability to detect and manipulate chemicals in a handedness-selective manner is highly relevant for drug synthesis, amongst other things.

Light can be used as a tool for such purposes, and optimized nanophotonic structures can help shape chiral light–matter-interactions, that is, the structures' interaction with light of different circular polarization handedness (or helicity). In particular, the chiral nature of an object's response to light has been formalized as electromagnetic chirality (em-chirality) in [1].

Starting from scatterers optimized for a strongly em-chiral response, we describe how to construct metasurfaces to act as chiral mirrors. Then, we study the possibilities of combining these metasurfaces to build chiral cavities. Their purpose is to provide a strongly helicity-selective electromagnetic environment to enhance the enantioselectivity of chemical processes.

References

[1] I. Fernandez-Corbaton, M. Fruhnert, and C. Rockstuhl, Objects of Maximum Electromagnetic Chirality, Physical Review X **6** (2016), 031013.

MS17 **Wednesday, 15:00–15:25, SR -1.025**

Coupled harmonics in time-modulated wave systems

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When materials whose properties are modulated in time are excited with an incident wave of a single frequency, the scattered field consists of a family of coupled harmonics at frequencies differing by the frequency of temporal modulation. Similarly, the temporal modulation induces coupling between the resonance frequencies, leading to exceptional points at certain modulation amplitudes. Moreover, the lack of energy conservation causes scattering coefficients to blow up when (complex) resonances cross the real axis. We have developed an integral operator approach to characterise the resonance and scattering properties of a collection of scatterers whose properties are modulated in time. We use this integral framework to develop efficient numerical schemes and to derive small-volume asymptotic formulas (analogous to the classical results for static scatterers).

References

[1] E. O. Hiltunen and B. Davies, Coupled harmonics due to time-modulated point scatterers. *Physical Review B*, in press (2024).

MS17 **Wednesday, 15:30–15:55, SR -1.025**

TDG methods for scattering by periodic structures

Armando Maria Monforte[1,*], **Andrea Moiola**[1]
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We consider the diffraction of time-harmonic waves incident on a periodic grating. Periodic structures play a pivotal role in various engineering and scientific applications, including antenna design, metamaterial characterization, and photonic crystal analysis.

We focus on the scattering of electromagnetic waves by planar gratings, modelled by the 2D Helmholtz equation with variable relative permittivity ε, and suitable boundary and radiation conditions. We introduce a Trefftz Discontinuous Galerkin (TDG) approach to solve the problem.

The TDG method is a powerful approach for analyzing scattering by periodic structures, since it offers an efficient strategy to reduce computational complexity. We discuss key concepts such as Trefftz functions (i.e. local Helmholtz solutions), DG formulations, exact integration rules, the treatment of quasi-periodic boundary conditions, and Dirichlet-to-Neumann operators to deal with radiation conditions.

We introduce the new DtN-TDG method and prove some good properties such as well-posedness. We study explicit stability estimates for the continuous problem and the TDG scheme.

Lastly, we provide numerical examples to illustrate the effectiveness and versatility of the TDG method, using plane waves as basis functions.

MS17 **Wednesday, 16:30–16:55, SR -1.025**

A high-accuracy mode solver for acoustic scattering by a periodic array of axially symmetric obstacles

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We are concerned with guided modes of an acoustic wave propagation problem on a periodic array of axially symmetric obstacles. A guided mode refers to a quasi-periodic eigenfield that propagates along the obstacles but decays exponentially away from them in the absence of incidences. Thus, the problem can be studied in an unbound unit cell due to the quasi-periodicity. We truncate the unit cell onto a cylinder enclosing the interior obstacle in terms of utilizing Rayleigh's expansion to design an exact condition on the lateral boundary. We derive a new boundary integral equation (BIE) only involving the free-space Green function on the boundary of each homogeneous region within the cylinder. Due to the axial symmetry of the boundaries, each BIE is decoupled via the Fourier transform to curve BIEs and they are discretized with high-accuracy quadratures. With the lateral boundary condition and the side quasi-periodic condition, the discretized BIEs lead to a homogeneous linear system governing the propagation constant of a guided mode at a given frequency. The propagation constant is determined by enforcing that the coefficient matrix is singular. The accuracy of the proposed method is demonstrated by a number of examples even when the obstacles have sharp edges or corners.

MS17 **Wednesday, 17:00–17:25, SR -1.025**

Direct and Inverse Scattering Problems for Locally Perturbed Periodic Structures

Andreas Kirsch[1,*]

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In this talk we will study the scattering of time-harmonic point sources or plane waves in some domain of the plane which is a local perturbation of a domain D where D is periodic with respect to the horizontal axis and bounded with respect to the vertical axis. The main part of the talk is concerned with the discussion of a suitable radiation condition for this scattering problem and present results for uniqueness and existence. Then we turn to the inverse problem to determine properties of the perturbation from the measured fields and prove uniqueness of some related inverse problems.

MS17	Wednesday, 17:30–17:55, SR -1.025

Convergence of the PML method for the biharmonic wave scattering problem in periodic structures

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We consider the scattering of biharmonic waves by a one-dimensional periodic array of cavities embedded in an infinite elastic thin plate. The transparent boundary conditions are introduced to formulate the problem from an unbounded domain to a bounded one. The well-posedness of the associated variational problem is demonstrated utilizing the Fredholm alternative theorem. The perfectly matched layer (PML) method is employed to reformulate the original scattering problem, transforming it from an unbounded domain to a bounded one. The transparent boundary conditions for the PML problem are deduced, and the well-posedness of its variational problem is established. Moreover, exponential convergence is achieved between the solution of the PML problem and that of the original scattering problem.

Minisymposium 18

Recent Methods in Nonlinear Dispersive Waves

Organizers: Mihaela Ifrim and Daniel Tătaru

MS18 **Monday, 11:45–12:10, SR -1.025**

Low regularity global solutions for quasilinear wave equations

Albert Ai[1,*], **Mihaela Ifrim**[2]

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We consider the quasilinear wave equation, where the nonlinearity consists of linear combinations of the classical quadratic null forms. The global existence and asymptotics of small data solutions has recently been an active area of research, in both the two and three dimensional settings.

On the other hand, recent work by Ifrim-Stingo [1] on the two dimensional nonlinear wave-Klein-Gordon (WKG) system gives the almost-global existence of solutions to the WKG system for initial data under limited decay and regularity assumptions. The work combines several innovations to achieve this result, most notably a dyadic decomposition of spacetime adapted to the light cone, previously used by Metcalfe-Tataru-Tohaneanu; as well as Alinhac's ghost weight method to localize tangential energy estimates.

Our current interest is to establish global solutions for the two dimensional quasilinear wave equation for small non-compactly supported data, with minimal decay and regularity assumptions. In particular, we attempt to establish bounds using just one vector field.

References

[1] M. Ifrim and A. Stingo, Almost global well-posedness for quasilinear strongly coupled wave-Klein–Gordon systems in two space dimension, *preprint* 2019, arXiv 1910.12673.

MS18 Monday, 12:15–12:40, SR -1.025

Global Dynamics of small data solutions to the Derivative Nonlinear Schrödinger equation

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In this paper, we consider the derivative nonlinear Schrödinger (DNLS) equation. Recently, global well-posedness in the critical scaling space of $L^2(\mathbb{R})$ was proven for (DNLS) [1]. While the existence theory has been extremely well studied, properties like dispersive estimates for the solutions have not yet been investigated. Here we address this question for the problem with small and localized data, and show that a dispersive estimate for the solution holds globally in time. For the proof of our result we use vector field methods combined with the *testing by wave packets method* of Ifrim and Tataru [2], whose implementation in this problem is novel.

References

[1] B. Harrop-Griffiths, R. Killip, M. Ntekoume and M. Vişan, Global well-posedness for the derivative nonlinear Schrödinger equation in $L^2(\mathbb{R})$, *Preprint available at https://arxiv.org/pdf/2204.12548.pdf*: 2022.

[2] M. Ifrim and D. Tataru, Global bounds for the cubic nonlinear Schrödinger equation (NLS) in one space dimension, *Nonlinearity*, 28(8): 2015 pp. 2661-2675.

MS18 Monday, 12:45–13:10, SR -1.025

Bifurcation structure of time-periodic solutions to nonlinear wave equations

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We study time-periodic solutions for nonlinear wave equations on an interval with Dirichlet boundary conditions and find intricate bifurcation structures forming fractal-like patterns [1]. We propose a systematic approach to analysing these complex structures [2]. Our results complement the classic proofs of existence of small amplitude periodic solutions with frequencies belonging to nowhere dense sets, see for example [3,4]. In particular, they suggest that the mentioned gaps in frequencies are filled with large amplitude solutions.

References

[1] F. Ficek and M. Maliborski, Periodic solutions for the 1d cubic wave equation with Dirichlet boundary conditions, arXiv:2407.16507.

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[4] M. Berti and P. Bolle, Cantor families of periodic solutions for completely resonant nonlinear wave equations, *Duke Mathematical Journal* **134** (2006), pp. 359–419.

MS18 **Monday, 15:00–15:25, SR -1.025**

Low regularity well-posedness for the Generalized Surface Quasi-Geostrophic front equation

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We consider the well-posedness of the generalized surface quasi-geostrophic (gSQG) front equation. By making use of the null structure of the equation, we carry out a paradifferential normal form analysis in order to obtain balanced energy estimates, which allows us to prove the local well-posedness of the g-SQG front equation in the non-periodic case at a low level of regularity (in the SQG case, this is only one half of a derivative above scaling). In addition, we establish global well-posedness for small and localized rough initial data, as well as modified scattering, by using the testing by wave packet approach of Ifrim-Tataru.

This is joint work with Albert Ai.

MS18 **Monday, 15:30–15:55, SR -1.025**

Modified scattering for the three dimensional Maxwell-Dirac system

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In this talk, I will present our recent result on global well-posedness and modified scattering for the massive Maxwell-Dirac system in the Lorenz gauge in $\mathbb{R}^{1+3}$ for small, sufficiently smooth and decaying initial data. Our approach both exploits the close connection of the massive Maxwell-Dirac system with the wave-Klein-Gordon equations and specific structural properties of the Dirac equation. The modified scattering result follows from a precise description of the asymptotic behavior of the solution inside the light cone, which we derive via the method of testing with wave packets of Ifrim-Tataru.

References

[1] S. Herr, M. Ifrim and M. Spitz, Modified scattering for the three dimensional Maxwell-Dirac system, arXiv:2406.02460.

MS18 **Monday, 16:30–16:55, SR -1.025**

Low Regularity Solutions for the General Quasilinear Ultrahyperbolic Schrödinger Equation

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I will present a novel method for establishing large data local well-posedness in low regularity Sobolev spaces for general quasilinear Schrödinger equations with non-degenerate and nontrapping metrics. Our result represents a substantial improvement over the landmark results of Kenig, Ponce, Rolvung and Vega, as it weakens the regularity and decay assumptions to the same scale of spaces considered in a recent paper of Marzuola, Metcalfe, and Tataru, but removes the uniform ellipticity assumption on the metric from their result. Our method has the additional benefit of being relatively simple but also very robust. In particular, it only relies on the use of pseudodifferential calculus for classical symbols. This is joint work with Mitchell Taylor.

MS18 **Monday, 17:00–17:25, SR -1.025**

Justification of the Benjamin-Ono equation as an internal water waves model

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The Benjamin-Ono (BO) equation is a nonlocal asymptotic model for the unidirectional propagation of weakly nonlinear, long internal waves in a two-layer fluid. The equation was introduced formally by Benjamin in the '60s and has been a source of active research since. For instance, the study of the long-time behavior of solutions, stability of traveling waves, and the low regularity well-posedness of the initial value problem. However, despite the rich theory for the BO equation, it was an open question whether its solutions are close to the ones of the original physical system.

In this talk, I will explain the main steps involved in the rigorous derivation of the BO equation.

MS18 **Monday, 17:30–17:55, SR -1.025**

Sharp well-posedness for the free boundary Euler equations

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We will discuss a recent preprint in which we establish an optimal local well-posedness theory in H^s based Sobolev spaces for the free boundary incompressible Euler equations on a connected fluid domain. Some components of this result include: (i) Local well-posedness in the Hadamard sense, i.e., local existence, uniqueness, and the first proof of continuous dependence on the data, all in low regularity Sobolev spaces; (ii) Enhanced uniqueness: A uniqueness result which holds at the level of the Lipschitz norm of the velocity and the $C^{1,\frac{1}{2}}$ regularity of the free surface; (iii) Stability bounds: We construct a nonlinear functional which measures, in a suitable sense, the distance between two solutions (even when defined on different domains) and we show that this distance is propagated by the flow; (iv) Energy estimates: We prove essentially scale invariant energy estimates for solutions, relying on a newly constructed family of refined elliptic estimates; (v) Continuation criterion: We give the first proof of a continuation criterion at the same scale as the classical Beale-Kato-Majda criterion for the incompressible Euler equations on fixed domains. Roughly speaking, we show that solutions can be continued as long as the velocity is in $L^1_T W^{1,\infty}$ and the free surface is in $L^1_T C^{1,\frac{1}{2}}$; (vi) A novel proof of the construction of regular solutions.

Our entire approach is in the Eulerian framework and can be adapted to work in relatively general fluid domains. This is based on joint work with Mihaela Ifrim, Ben Pineau and Daniel Tataru.

MS18 **Monday, 18:00–18:25, SR -1.025**

The Cauchy problem for the periodic KP-II equation: revisited

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30 years ago, Bourgain [1] showed global well-posedness of the KP-II equation in $L^2(\mathbb{T}^2)$:

$$\begin{cases} \partial_t u + \partial_x^3 u - \partial_x^{-1}\partial_y^2 u &= u\partial_x u, \quad (t,x,y) \in \mathbb{R}\times\mathbb{T}^2, \\ u(0) &= u_0 \in L^2(\mathbb{T}^2). \end{cases}$$

We extend his result to Sobolev spaces of negative order $H^{s,0}(\mathbb{T}^2)$, $s > -\frac{1}{90}$, which is the first improvement since [1]. To this end, we combine Strichartz estimates derived from a new ℓ^2-decoupling inequality due to Guth–Maldague–Oh [2] with short-time Fourier restriction [3].

References

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Minisymposium 19

Spectral Theory and Applications

Organizers: Illia Karabash, Konstantin Pankrashkin, and Olaf Post

MS19	Monday, 11:45–12:10, SR 2.067

The one-dimensional Dirac equation: extension theory and boundary conditions

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Let $\mathcal{G}$ be a finite metric graph. A full characterization of the self-adjoint realizations of the Dirac equation on $L^2(\mathcal{G})$ has been known since a pioneering paper by Bolte and Harrison in 2002. I will present a different formalism yielding an analogous characterization on weighted L^2-spaces: in this way, it is possible to characterize evolution driven by unitary groups or, more generally, by contractive semigroups.

If time allows, I will present recent results concerning spectral symmetry issues for the Dirac equation with MIT bag conditions.

This is based on joint work with Catherine Drysdale, Markus Holzmann, Marjeta Kramar-Fijavž and Serge Nicaise.

MS19 Monday, 12:15–12:40, SR 2.067

Magnetic Bernstein inequalities and spectral inequality on thick sets for the Landau operator

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We prove a spectral inequality for the Landau operator. This means that for all f in the spectral subspace corresponding to energies up to E, the L^2-integral over suitable $S \subset \mathbb{R}^2$ can be lower bounded by an explicit constant times the L^2-norm of f itself. We identify the class of all measurable sets $S \subset \mathbb{R}^2$ for which such an inequality can hold, namely so-called *thick* or *relatively dense* sets, and deduce an asymptotically optimal expression for the constant in terms of the energy, the magnetic field strength and in terms of parameters determining the thick set S. Our proofs rely on so-called magnetic Bernstein inequalities.

As a consequence, we obtain the first proof of null-controllability for the magnetic heat equation (with sharp bound on the control cost), and can relax assumptions in existing proofs of Anderson localization in the continuum alloy-type model.

References

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MS19 Monday, 12:45–13:10, SR 2.067

Dispersive estimates for the exterior ball problem for Maxwell's equations.

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The Maxwell scattering problem for conducting spheres is of considerable interest in engineering applications. Recently it was shown [3] that the electric field as modelled by Maxwell's equations on $M = \mathbb{R}^3 \setminus \mathcal{O}$, where $\mathcal{O}$ is a ball gives a different scattering rate than the usual $L^1 \to L^\infty$ dispersive estimate for the corresponding acoustic scattering problem [2]. However it is not known whether or not this result is sharp in terms of powers of h. We aim to show through Mie scattering that any progress on the Maxwell scattering problem for the electric field can be modelled by the acoustic scattering problem with *Neumann* boundary conditions. The key point is the identification of the spectral theory for the Neumann problem with that of the magnetic field, using [1]. The functional calculus for the Neumann and Dirichlet problem for evolution operators can be linked, and this result can be shown to be sharp.

References

[1] A. Strohmaier and A. Waters. Geometric and obstacle scattering at low energy. *Comm. Partial. Diff. Eq.*, 45(11): 1451-1511, 2020.

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[3] Y.L. Fang and A. Waters Dispersive estimates for Maxwell's equations in the exterior of a sphere Journal of Diff. Eq., 415 2025.

MS19	Monday, 16:30–16:55, SR 2.067

Magnetic Neumann Laplacian on a planar exterior domain in the weak field limit

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The central object of the talk is the magnetic Neumann Laplacian with homogeneous magnetic field in the complement of a bounded simply-connected smooth domain in the plane. The essential spectrum of this operator consists of the Landau levels, to which the discrete eigenvalues accumulate from below. We analyse the exterior of the disk in detail, observing that the ground-state eigenfunction is not radial and finding the accurate asymptotics of the low-lying eigenvalues in the weak field limit. We expect that the exterior of the disk is a global maximizer of the lowest magnetic Neumann eigenvalue within the class of complements of bounded simply-connected smooth domains of fixed area. In this talk, we will present a partial result in this direction, where the fixed-area constraint is substituted by a weaker constraint in terms of p-moments of the domain. Motivated by this eigenvalue optimization question, we also establish an asymptotic upper bound in the weak field limit on the lowest magnetic Neumann eigenvalue of the exterior of a bounded star-shaped domain.

MS19	Monday, 17:00–17:25, SR 2.067

Uniform resolvent estimates for subwavelength resonators

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Subwavelength resonators are small scaled objects that exhibit contrasting medium properties (eigher in intensity or sign) while compared to the ones of a uniform background. Such contrasts allow them to resonate at specific frequencies. There are two ways to mathematically define these resonances. First, as the frequencies for which the related system of integral equations is not injective. Second, as the frequencies for which the related resolvent operator of the natural Hamiltonian, given by the wave-operator, has a pole.

In this work, we consider, as the subwavelength resonator, the Minneart bubble. We show that these two mentioned definitions are equivalent. Most importantly,

- We derive the related resolvent estimates which are uniform in terms of the size/contrast of the subwavelength resonator. As a by product, we show that the resolvent operators have no resonances in the upper half complex plane while they exhibit two resonances in the lower half plane which converge to the real axis, as the size of the bubble tends to zero. As these resonances are poles of the natural Hamiltonian, given by the wave-operator, and have the Minnaert frequency as their dominating real part, this justifies calling them Minnaert resonances.
- We derive the asymptotic estimates of the generated scattered fields which are uniform in terms of the incident frequency and which are valid everywhere in space (i.e. inside or outside the bubble).

The dominating parts, for both the resolvent operator and the scattered fields, are given by the ones of the point-scatterer supported at the location of the bubble. In particular, these dominant parts are non trivial (not the same as those of the background medium) if and only if the used incident frequency identifies with the Minnaert one.

MS19 **Monday, 17:30–17:55, SR 2.067**

Capacity and type of state in non-Archimedean graphs

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The physical quantity capacitance describes the capacity of a capacitor to store an electric charge. In spectral graph theory a capacity of a vertex in graph characterizes the current flow in the corresponding electrical network. Moreover, positivity of the capacity is closely related to the spectrum of the discrete Laplacian and to the type of the underlying Markov chain. In this talk we consider graphs, whose edge weights belong to a non-Archimedean ordered field. We introduce the capacity for vertices of such graphs and investigate properties of this quantity, including its positivity. Moreover, we relate graphs over non-Archimedean ordered fields to discrete Markov chains. In particular, we give a characterization of the Markov chains which can arise in such a way. Finally, we present a characterization for recurrence and transience in terms of a quantity related to the non-Archimedean capacity.

References

[1] Matthias Keller, Anna Muranova. *Recurrence and transience for non-Archimedean and directed graphs.* arXiv preprint: https://arxiv.org/abs/2406.17344, June 2024.

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MS19 **Monday, 18:00–18:25, SR 2.067**

Semiclassical Analysis of Coastal-Trapped Waves: Asymptotic Behavior in Nearly Integrable Case

Vladislav Rykhlov[1,*], **Anatoly Anikin**[2]
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We construct semiclassical spectral series, including asymptotic eigenvalues and formal asymptotic eigenfunctions (quasimodes), for the operator $\langle \nabla, D(x)\nabla \rangle$ in $\Omega \subset \mathbb{R}^2$ with $D(x)$ vanishing on the boundary $\partial\Omega$. The operator is defined on $L^2(\Omega)$ as a Friedrichs self-adjoint extension of the minimal operator given by the same differential expression. We refine the results of [1] by considering a nearly integrable case using the uniformization approach developed by V. E. Nazaikinskii with collaborators, idea of which is to remove the degeneracy by lifting the operator to a higher-dimensional configuration space. Our method provides explicit formulas for quasimodes in terms of Airy and Bessel functions, suitable for practical computations, e.g., in Wolfram Mathematica, and can be applied to other problems as well. The talk is based on paper [2].

References

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[2] A. Yu. Anikin, V. V. Rykhlov, High-Frequency Two-Dimensional Asymptotic Standing Coastal-Trapped Waves in Nearly Integrable Case, *preprint on arXiv:*2406.11000 (2024).

MS19 **Tuesday, 11:30–11:55, SR 2.067**

Limiting absorption principle for a degenerate boundary-value problem modeling plasma resonances

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In this work we study a boundary-value problem (BVP) for a degenerate PDE $\operatorname{div}(\alpha\nabla u) + \omega^2 u = f$, where α is a coefficient changing its sign smoothly across an interface inside the domain. This model stems from plasma physics, where singular solutions to the above problem are known to be responsible for the plasma heating phenomena.

We study the viscosity limit for the above problem, defined via the family of solutions to regularized BVPs for $\operatorname{div}((\alpha + i\nu)\nabla u^\nu) + \omega^2 u^\nu = f$. We prove that, apart from a discrete set of ω^2, as $\nu \to 0+$, the $L^2 - \lim\limits_{\nu\to 0+} u^\nu$ is well-defined, and can be decomposed into a regular and singular parts. This decomposition allows to establish a non-standard Green's formula for the limiting problem. To define a well-posed problem satisfied by the limiting solution, we introduce a radiation-like condition into the domain of the underlying operator. This new operator is non-self-adjoint and has a compact resolvent. Time permits, we discuss the location of its spectrum and localization properties of the eigenfunctions.

References

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MS19 **Tuesday, 12:00–12:25, SR 2.067**

Quantum Tunneling and the Aharonov-Bohm effect

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We investigate a Hamiltonian with radial potential wells and an Aharonov-Bohm vector potential with two poles. Assuming that the potential wells are symmetric, we derive the semiclassical asymptotics of the splitting between the ground and second state energies. We observe flux effects due to the Aharonov-Bohm vector potential that are of lower order compared to the contributions coming from the potential wells.

MS19 **Tuesday, 12:30–12:55, SR 2.067**

Atypical Spectra and Dynamics of Non-Locally Finite Crystals

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We investigate the spectral theory of periodic graphs which are not locally finite but carry non-negative, symmetric and summable edge weights. These periodic graphs are shown to have rather intriguing behaviour. We construct a periodic graph whose Laplacian has purely singular continuous spectrum. We prove that motion remains ballistic along at least one layer under quite general assumptions. We construct a graph whose Laplacian has purely absolutely continuous spectrum, exhibits ballistic transport, yet fails to satisfy a dispersive estimate. This answers negatively an open question in this regard, in our setting. Concerning point spectrum, we construct a graph with a partly flat band whose eigenvectors must have infinite support, in contrast to the locally finite case. We believe the present class of graphs can serve as a playground to better understand exotic spectra and dynamics in the future.

Based on joint work with Joachim Kerner, Olaf Post and Matthias Täufer.

Minisymposium 20

Nonlocal Dispersive Equations

Organizers: Christian Klein and Nikola Stoilov

MS20	Thursday, 15:00–15:25, SR -1.025

Spectral approaches to solutions that are non-trivial at infinity

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We present several methods for numerically finding solutions to partial differential equations as well as fractional differential equations, that have non-trivial behaviour at infinity. Generally, the compactified real line is divided into a number of intervals, thus amounting to a multidomain approach; after transformations ensuring analyticity of the respective integrands, the integrals over the different domains are computed with a Clenshaw–Curtis algorithm. As examples, we consider solutions the NLS and KdV as well as solitary waves for fractional Korteweg-de Vries equations. [1] , On numerical approaches to nonlinear Schr¨odinger and Ko- rteweg–de Vries equations for piecewise smooth and slowly decaying initial data, Phys. D 454, (2023), 133885 [2] . (2024), 1–23.

References

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[2] C. Klein, N. Stoilov, Multidomain spectral approach to rational-order fractional derivatives *Studies of Applied Mathematics* (2024), 1-23

MS20 **Thursday, 15:30–15:55, SR -1.025**

The Fourier spectral approach to the spatial discretization of Whitham–Boussinesq models

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We will discuss the rigorous justification of the numerical spatial discretization by means of Fourier spectral methods of Whitham–Boussinesq equations modelling the dynamical evolution of surface gravity waves. Consider systems of the form

$$\begin{cases} \partial_t\zeta + \mathsf{F}_1^\mu \nabla\cdot\mathbf{v} + \mathsf{F}_2^\mu\nabla\cdot(\zeta(\mathsf{F}_2^\mu\mathbf{v})) = 0, \\ \partial_t\mathbf{v} + \nabla\zeta + ((\mathsf{F}_2^\mu\mathbf{v})\cdot\nabla)(\mathsf{F}_2^\mu\mathbf{v}) = 0, \end{cases}$$

where F_1^μ and F_2^μ are suitable Fourier multipliers depending on a parameter μ (possibly small) and such that $\mathsf{F}_1^\mu|_{\mu=0} = \mathsf{F}_2^\mu|_{\mu=0} = \mathrm{Id}$. The standard Fourier spectral method for spatial discretization amounts to considering the modified system

$$\begin{cases} \partial_t\zeta + \mathsf{F}_1^\mu \nabla\cdot\mathbf{v} + \Pi_N\mathsf{F}_2^\mu\nabla\cdot(\zeta(\mathsf{F}_2^\mu\mathbf{v})) = 0, \\ \partial_t\mathbf{v} + \nabla\zeta + \Pi_N((\mathsf{F}_2^\mu\mathbf{v})\cdot\nabla)(\mathsf{F}_2^\mu\mathbf{v}) = 0, \end{cases}$$

where Π_N is a low pass filter for wavenumbers $|\mathbf{k}| \le N$.

We shall discuss stability and convergence results as $N \to \infty$, depending on the size of the parameter μ and the (semilinear, quasilinear) structure of the system.

MS20 **Thursday, 16:30–16:55, SR -1.025**

Nonlinear transverse instability of line periodic waves

Wei Lian[1], **Erik Wahlén**[1,*]
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Periodic travelling wave solutions of the KdV equation can also be seen as line periodic solutions of the Kadomtsev–Petviashvili (KP) equation in two space dimensions. While the periodic waves are known to be orbitally stable under the KdV flow with respect to co-periodic or subharmonic perturbations, they have been shown to be spectrally unstable as solutions to the KP-I equation (e.g. modelling water waves with strong surface tension). The passage from spectral to nonlinear instability is however not automatic for dispersive PDE. While a lot of progress has been made on this topic for line solitary waves, the periodic setting is far less explored. In my talk I will present a recent result which allows one to pass from spectral to nonlinear transverse instability of line periodic waves for a general class of nonlinear dispersive equations. This result applies to KP-I, but also some nonlocal equations.

MS20 Thursday, 17:00–17:25, SR -1.025

Asymmetric capillary–gravity water waves in the steady periodic setting

Mats Ehrnström[1,*], **Boris Buffoni**[2], **Douglas Svensson Seth**[3]

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We discuss ongoing work on asymmetric capillary–gravity surface waves in the Euler equations. It has been known for a long time that the setting of weak surface tension allows for higher-dimensional bifurcation from still water, giving rise to multimodal waves with more than one crest in a period. These waves have, however, all been symmetric, although numerical calculations indicate the presence of truly asymmetric waves in the steady periodic setting. Recently, Mæhlen and Svensson Seth extended earlier bifurcation results for the gravity–capillary Whitham equation, showing that asymmetric solutions exist as natural extensions of bimodal waves. In this work, which is joint with Douglas Svensson Seth (NTNU) and Boris Buffoni (EPFL), we investigate the existence of such asymmetric solutions in the Euler equations.

Minisymposium 21

Kerr Frequency Combs — From Models to Experiments and Back

Organizers: Christian Koos and Wolfgang Reichel

MS21	Monday, 11:45–12:10, SR 3.068

Modeling Quadratic Optical Frequency Combs

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Optical frequency combs can be generated from the modulational instability (MI) of a continuous wave (CW) pump that is injected in a passive cavity with third-order nonlinearity. Although Kerr combs have been successfully demonstrated in the context of various applications, from metrology to spectroscopy and telecommunications, there are constraints that limit their practical use. Specifically, anomalous group-velocity dispersion is required at the pump wavelength; moreover, comb generation is difficult in spectral regions that lack stable intense CW pump lasers, such as the visible and mid-infrared. Finally, the weak cubic nonlinearity sets a relatively high pump power threshold for comb generation.

In recent years, a different approach to comb generation has been introduced, based on MI in a second-order nonlinear cavity. In this talk, we overview the theory and experimental progress in the demonstration of such quadratic combs. For quadratic comb generation, there are two main experimental configurations. The first is based on cavity second harmonic generation (SHG): combs are generated around both the CW pump fundamental frequency (FF), and around its second harmonic (SH). The second configuration involves degenerate optical parametric oscillation (OPO), where combs are generated around the pump FF and its subharmonic frequency. Phase coherence among the comb teeth is of paramount importance for applications: it is ensured by the generation of temporal cavity solitons. Remarkably, it has been predicted that coupled bright and dark soliton combs can be generated around the FF and the SH in quadratic nonlinear cavities.

MS21	Monday, 12:15–12:40, SR 3.068

Uniform Stability to Subharmonic Perturbations in the Lugiato-Lefever Equation

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We study the linear and nonlinear dynamics of spectrally-stable T-periodic stationary solutions of the Lugiato-Lefever equation (LLE), a damped nonlinear Schrödinger equation with forcing that arises in nonlinear optics. Spectrally-stable T-periodic solutions are known to be nonlinearly stable to NT-periodic, i.e., subharmonic, perturbations for each $N \in \mathbb{N}$ with exponential decay rates of the form $e^{-\delta_N t}$. However, both the exponential rates of decay δ_N and the allowable size of initial perturbations tend to 0 as $N \to \infty$ that this result is non-uniform in N and is in fact empty in the limit $N = \infty$.

The primary goal of this talk is to introduce a methodology, in the context of the LLE, by which a uniform (in N) stability result for subharmonic perturbations may be achieved at both the linear and nonlinear level. The obtained uniform decay rates are shown to agree precisely with the polynomial decay rates of localized, i.e., integrable on the real line, perturbations of such spectrally-stable periodic solutions of the LLE.

MS21	Monday, 15:00–15:25, SR 3.068

RF-Synchronized Kerr Frequency Combs for Optical Arbitrary Waveform Generation

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Chip-scale Kerr soliton microcombs contain tens or even hundreds of mutually coherent narrow-linewidth continuous-wave (CW) carriers with large comb line spacings of tens or even hundreds of gigahertz, thus serving as superior multi-wavelength light sources for a wide range of applications. Recently, we have demonstrated a series of ultra-broadband signal acquisition systems, such as optical arbitrary waveform measurement (OAWM) [1] and photonic-electronic analogue-to-digital conversion (PE-ADC) [2], that rely on free-running Kerr soliton microcombs, for which the Kerr comb line spacings were not synchronized to the clocks of the associated electronic digital signal processors. This leads to unavoidable phase and frequency drifts between electronic and photonic subsystems, which can be compensated for by dedicated digital signal processing (DSP) of the acquired waveforms. However, spectral stitching based

MS21	Monday, 15:30–15:55, SR 3.068

Pinning in an extended Lugiato-Lefever model

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We consider pinning phenomena of Kerr soliton combs within a microresonator driven by dual mode pumping. This system can be modeled by a new variant of the Lugiato-Lefever equation:

$$\mathrm{i}u_t = -du_{xx} + \mathrm{i}V(x)u_x + (\zeta - \mathrm{i})u - |u|^2u + \mathrm{i}f,$$

which is a damped and driven nonlinear Schrödinger equation with a periodic potential $V(x)$. In the first part, we discuss the existence of nontrivial stationary 2π-periodic solutions using bifurcation theory. We show that localized solitons exist when the potential $V(x)$ has a sign change, with their intensity maxima pinned to a zero of $V(x)$. In the second part, we discuss the stability properties of these solutions and present numerical simulations with `pde2path`, which complement our analytical findings.

This is based on joint work with Dmitry Pelinovsky (McMaster University) and Wolfgang Reichel (KIT).

MS21	Tuesday, 15:00–15:25, SR 3.068

Dynamics of Kerr frequency combs and temporal localized states in time-delayed micro-cavities

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We discuss the existence of a novel type of temporal localized structure in injected Kerr-Gires-Tournois interferometers. These bright pulses exist both in the normal and anomalous dispersion regimes, yet they do not correspond to the usual scenario of domain wall locking or cavity solitons formation. The new states are observed beyond the mean-field limit and out of the bistable region. Their shape is uniquely defined, with peak intensities beyond that of the upper steady state, and they are stable over a broad range of the injection field, highlighting their potential for optical frequency comb generation.

References

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MS21	Tuesday, 15:30–15:55, SR 3.068

Kerr combs in nano-structured microresonators

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Microresonator Kerr combs, as compact sources of high-repetition-rate waveforms, have established themselves as a transformative photonic technology [1]. They arise through nonlinear optical self-organization and their dynamics are largely determined by the sign and magnitude of the resonator's group velocity dispersion. Recent advances in subwavelength nanostructuring of microresonators make it possible to modify the resonant modes on a resonance-by-resonance basis. We will explore how these modifications affect the energy flow within nonlinear microresonators and how they enable the design of novel systems with reduced operational complexity, improved stability, and potential for new functionality [2,3].

References

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Minisymposium 22

Long Wave Approximations in Homogeneous, Periodic and Stochastic Media

Organizers: Xian Liao and Guido Schneider

MS22	**Tuesday, 11:30–11:55, SR 2.066**

Korteweg-De Vries (KdV) Approximation for Certain Random Fermi-Pasta-Ulam-Tsingou (FPUT) Lattices

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We will discuss a recent result regarding the KdV approximation of certain random FPUT lattices. To highlight some of the techniques and challenges involved in the proof of such an approximation, we will also discuss the homogenization of a one-dimensional and two-dimensional linear lattice with random masses. Then we will see how the large time scales needed for the KdV approximation creates difficulties in estimating error terms that fall out of the long wave ansatz. These difficulties may arise because it may be that only certain random FPUT lattices have the usual KdV approximation. Specifically, we require the random masses (or springs) to be two discrete derivatives of a doubly infinite sequence of i.i.d. random variables. This technical condition, which we call *transparency*, was inspired by a similar condition used in another setting to formally derive a KdV approximation [1]. We will discuss how an auto-regressive process can be used to prove such an approximation for the FPUT lattice by controlling stochastic fluctuations arising in the error. Finally, we present numerical evidence suggesting a KdV approximation of a lattice with i.i.d. masses is likely not possible.

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References

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MS22 | **Tuesday, 12:00–12:25, SR 2.066**

Approximation of (some) random FPUT lattices by KdV equations

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We consider a Fermi–Pasta–Ulam–Tsingou lattice with randomly varying coefficients. We discover a relatively simple condition which when placed on the nature of the randomness allows us to prove that small amplitude/long wavelength solutions are almost surely rigorously approximated by solutions of Korteweg–de Vries equations for very long times. The key ideas combine energy estimates with homogenization theory and the technical proof requires a novel application of autoregressive processes.

MS22 | **Tuesday, 12:30–12:55, SR 2.066**

On the Justification of Long Wave Approximations for General Classes of Quasilinear Dispersive Systems

Wolf-Patrick Düll[1,*], **Franz Schewe**[2]
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The rigorous justification of modulation equations like the KdV equation as approximation equations by error estimates in the physically relevant length and time scales is a highly nontrivial problem if the original dispersive systems are quasilinear. In this talk, we present a new and systematic procedure of constructing energies for the error estimates taking advantage of the Hamiltonian structure of the quasilinear terms of the underlying homogeneous or periodic systems.

MS22 **Tuesday, 16:30–16:55, SR 2.066**

Mechanical balance laws in the KdV Approximation

Henrik Kalisch[1,*]

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The KdV equation is a model for waves at the free surface of an incompressible and inviscid fluid. The equation is known to be an accurate model if the fluid bed is flat, transverse effects can be neglected and if the waves are long and of small amplitude when compared to the undisturbed depth of the fluid.

Mechanical balance laws play a pivotal role in fluid mechanics as they are a mathematical formulation of the basic physical principles of mass, momentum, and energy conservation. In connection with simplified model equations, mechanical balance laws be used for example to understand wave shoaling which is the process where ocean waves enter a zone of smaller depth, such as approaching a beach.

In this lecture, we will give an overview of recent results including applications to the study of surface waves [4] and some ideas on how to rigorously justify the balance laws associated to the KdV equations [1,2,3].

References

[1] A. Ali and H. Kalisch, On the formulation of mass, momentum and energy conservation in the KdV equation, *Acta Appl. Math.* **133** (2014), pp. 113–131.

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MS22 **Tuesday, 17:00–17:25, SR 2.066**

Mutidimensional modulation systems & large-time dynamics about periodic waves

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We illustrate on two examples how modulation systems — also called averaged systems or Whitham systems — may be used to describe the dynamics near periodic waves of multidimensional systems. In any case, the formal derivation of those systems may be obtained by a suitable variation on geometric optics.

Firstly we report from Audiard-Rodrigues (2022) how for planar waves of multidimensional Schrödinger equations the ill-posedness of the first-order modulation system at some wave parameter implies the spectral instability of the corresponding wave. Combined with expansions from Benzoni-Gavage-Mietka-Rodrigues (2021), this leads to a proof that essentially all small-amplitude periodic waves and all large-period periodic waves of Schrödinger equations are transversally unstable.

Secondly we report from Melinand-Rodrigues (2024) how for genuinely two-dimensional periodic waves of reaction-diffusion systems second-order modulation systems capture accurately the leading-order of nonlinear dynamics arising from small perturbations. The corresponding sharp results identify dispersive effects due to multidimensional effects and provides a partial multidimensional counterpart to the one-dimensional analysis from Johnson-Noble-Rodrigues-Zumbrun (2014).

MS22 Tuesday, 17:30–17:55, SR 2.066

Long-wave approximation of the NLS-hierarchy with non-zero boundary data by the KdV-hierarchy

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We prove a conjecture by F. Béthuel et al. relating the conserved quantities of the defocusing, one-dimensional Gross-Pitaevskii equation to those of the Korteweg-De Vries equation in a long-wave regime. Since a similar relation is present between the symplectic structures, we obtain formal and rigorous approximation results between the corresponding hierarchies.

References

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Minisymposium 23

Waveguides

Organizers: Ben Schweizer and Andreas Kirsch

MS23	Monday, 11:45–12:10, SR 1.067

The transfer matrix method in graphene-like acoustic networks

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Acoustic networks are made of waveguides connected on a graph. In the limit of slender tubes, the Helmholtz equation reduces to a discrete problem that exactly maps to a tight-binding Hamiltonian. If the considered graph is a hexagonal tiling, one obtains an acoustic analogue of graphene. This has been used to explore various configurations, and in particular the existence of edge modes with peculiar scattering properties [1,2].

In this work we discuss the transfer matrix formalism used to solve various scattering problems in this type of systems. The transfer matrix method in waveguide is usually tricky to apply, because the matrix is often singular. As we show, in acoustic networks, similar problems arise depending on the shape of the boundary. We discuss these issues and how to overcome them to solve scattering problems, and apply it to the case of topological edge modes scattering on impurities.

References

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MS23 **Monday, 12:15–12:40, SR 1.067**

Space time rays, caustics and interference pattern of the field in horizontally irregular waveguides

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We study a variant of well-known "vertical modes and horizontal rays" approach for acoustic waves propagation in shallow water. Our approach combines methods [1,2], existing in acoustics (wave equation with specific boundary conditions) and electrodynamics (modified wave equation with nonlocal integral operator). This allows to define space-time rays, related to the frequency-dependent properties of vertical modes as solutions of some Sturm-Liouville problem. Such rays allow us to describe propagation of short signals with changing frequency without use of standard methods like Fourier transform. Other important result is finding the relation between regular caustics (for long signals with constant frequency) and space-time caustics defined without such limitations.

References

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[2] Connor, K. A., Felsen, L. B. Complex space-time rays and their application to pulse propagation in lossy dispersive media. *Proceedings of the IEEE*, **62**(11), (1974), pp. 1586-1598.

MS23 **Monday, 12:45–13:10, SR 1.067**

Analysis of Space-Frequency Effects in Sound Propagation in Inhomogeneous Oceanic Waveguide Using Theory of Space-Time Rays

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Sound pulse propagation in shallow water waveguide with parameters varying in the horizontal plane and possibly in time is studied. It can be in the area of coastal wedge, underwater canyon, mount etc or in the presence of temperature front or nonlinear internal waves. Authors propose a theoretical technique which is some modification of the well-known approach [1], and which can be called vertical modes and space-time horizontal rays (STHR). Examples of constructing STHR in some typical situations (in particular in a wedge, underwater canyon, mount) are considered, some specific effects during signal propagation are analyzed, in particular, the so- called the pulse front tilt, previously discovered in laser optics [2] for short picosecond pulses passing through prism, and next was registered in underwater acoustics [3].

References

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MS23 **Monday, 15:00–15:25, SR 1.067**

Spectrum and pseudospectrum of reflctionless modes in acoustic waveguides

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We consider an infinite 2D acoustic waveguide with some local obstacle. A reflectionless frequency is such that there exists an acoustic field which is incoming on one side of the obstacle and outgoing on the other side. This means that for some particular incident wave, the obstacle does not produce any reflection, which may have many interesting applications.

From a mathematical point of view, using Perfectly Matched Layers with conjugate parameters on each side of the obstacle, the problem of reflectionless frequencies can be formulated as a linear but non-selfadjoint eigenvalue problem.

We are interested here by the convergence of the (complex) computed spectrum with respect to the length of the PMLs. In particular, we will illustrate some strange phenomena that can be explained by the notion of pseudospectrum.

References

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MS23 **Monday, 15:30–15:55, SR 1.067**

Mathematical and numerical analysis of the modes of a heterogeneous electromagnetic waveguide

A. Parigaux[1], A.-S. Bonnet-Ben Dhia[1], L. Chesnel[2], S. Fliss[1]
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[2]*IDEFIX (Inria - EDF - ENSTA Paris)*

We are interested in the design of transparent conditions in closed electromagnetic waveguides via a modal approach.

In the homogeneous case, i.e. with constant ε and μ, the modes (E_n, H_n, β_n) are easily obtained by solving scalar problems in the section S of the guide and are pairwise orthogonal in $\mathbf{L}^2(S)$. They are either propagating ($\beta \in \mathbb{R}$) or purely evanescent ($\beta \in i\mathbb{R}$) and they have phase and group velocities of the same sign.

For heterogeneous guides, i.e. with varying ε and μ in the section, these properties are generally not true and the mathematical analysis of the modes is much more delicate. In this talk, we present different formulations to study them and discuss their respective advantages.

For strong variations of ε and/or μ, we show numerically that inverse modes, with group and phase velocities of opposite sign, can exist. Such cases for which PMLs fail to capture the outgoing solution are one of the reasons why we develop modal transparent conditions.

References

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MS23 **Monday, 16:30–16:55, SR 1.067**

On the analysis of modes in a closed electromagnetic waveguide

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We consider the problem to find modal solutions $\mathcal{E}(t,\mathbf{x}) = \mathbf{E}(\hat{\mathbf{x}})e^{i\beta z - i\omega t}$, $\mathcal{H}(t,\mathbf{x}) = \mathbf{H}(\hat{\mathbf{x}})e^{i\beta z - i\omega t}$, $\beta \in \mathbb{C}, \omega \in \mathbb{R}$ to Maxwell's equations $\epsilon\partial_t\mathcal{E} = \operatorname{curl}\mathcal{H}$, $\mu\partial_t\mathcal{H} = -\operatorname{curl}\mathcal{E}$ in a closed $\mathbf{n}\times\mathcal{E} = 0$, $\mathbf{n}\times\operatorname{curl}\mathcal{H} = 0$ semi-infinite cylindrical waveguide $(\hat{\mathbf{x}}, z) \in \Omega\times\mathbb{R}^+$ with bounded cross section $\Omega \subset \mathbb{R}^2$. For homogeneous media $\epsilon = \text{const}, \mu = \text{const}$ it is well known that the this problem can be reduced to two separate Laplace eigenvalue problems with eigenvalues $\lambda_n^{\text{Dir/Neu}}$ and the wavenumbers satisfy $\beta_n^2 = \epsilon\mu\omega^2 - \lambda_n^{\text{Dir/Neu}}$. However, for heterogeneous media this problem becomes nonselfadjoint and its analysis is more challenging. In this talk we prove the existence and density of modes for $\epsilon, \mu \in W^{1,\infty}_{\text{piece-wise}}(\Omega)$ [1]. Our applied approach uses techniques as introduced in [2], which rest on a perturbation analysis of selfadjoint operators due to Keldysh [3, Appendix].

References

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MS23 **Monday, 17:00–17:25, SR 1.067**

Time-harmonic Maxwell's equations in half-waveguides

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Maxwell's equations are considered in a half-waveguide $\Omega_+ := \mathbb{R}_+ \times S \subset \mathbb{R}^3$ where $S \subset \mathbb{R}^2$ is a bounded Lipschitz domain in $\mathbb{R}^2$. The electric permittivity ε and the magnetic permeability μ are assumed to be strictly positive and periodic outside a compact set. Our Maxwell system is accompanied by a radiation condition that was introduced and investigated in [1]. We give a result on existence and uniqueness in the form of a Fredholm alternative: When there is no bound state, i.e., no non-trivial solution of the homogeneous problem on Ω_+, then there is a unique solution for every right hand side. Our approach is based on an energy method which was developed in [2] to study Helmholtz equations.

References

[1] A. Kirsch and B. Schweizer, Time harmonic Maxwell's equations in periodic waveguides, (submitted), TU Dortmund 2024-01

[2] B. Schweizer, Inhomogeneous Helmholtz equations in wave guides – existence and uniqueness results with energy methods in *European J. Appl. Math. Vol. 34(2),* pp. 211-237

MS23 **Monday, 17:30–17:55, SR 1.067**

Mathematical description of polarization

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We consider the time harmonic Maxwell equations in a complex geometry. We are interested in complex geometries that model pole filters or Faraday cages. We study the situation that the domain of interest contains inclusions with large or infinite conductivity, the inclusions are distributed in a periodic fashion along a surface. The periodicity is $\eta > 0$ and the shape of the inclusion depends also on η since we want to model, e.g., thin structures. We are interested in the limit $\eta \to 0$ and in effective equations. Depending on geometric properties of the inclusions, the effective system can imply polarisation or cancellation of the field.

References

[1] A. Lamacz-Keymling, B. Schweizer and D. Wiedemann, Mathematical description of polarization, *in preparation.*

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MS23 **Tuesday, 11:30–11:55, SR 1.067**

Dispersion curves for Bloch waves in an open periodic waveguide

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We consider the wave propagation in a periodic layer above a perfectly conducting plate and embedded in an isotropic and homogeneous background medium. In this open periodic waveguide, we investigate mathematical properties of dispersion curves, which together with the Bloch theory lead to spectra of the homogeneous scattering problem in a half space. Being different from a closed waveguide, the sesquilinear form in an open waveguide is not selfadjont and thus admits complex-valued dispersion curves. Using the perturbation theory for analytic Fredholm operators, we show existence, regularity and bifurcation theory of dispersion curves in an open waveguide. A sufficient condition is provided to ensure the discretness of real Bloch varieties above the light lines. Below the light lines there exist only real-valued Bloch varieties.

MS23 Tuesday, 12:00–12:25, SR 1.067

A novel method to analyse the radiation condition for the scattering problems with periodic media

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The radiation condition is the key topic in the analysis of scattering problems. Mathematically, a proper radiation condition guanratees the well-posedness of the problem; physically, a suitable radiation condition describes the physical process exactly. However, the topic is very challenging when the domain is periodic. Based on the upward propagation radiation condition (see, e.g., [1]), the authors in [2] have proved that the scattered field satisfies $\frac{\partial u}{\partial r} - \imath k u = o\left(r^{-1/2}\right)$. The result is extended to periodic inhomogeneous layers in [3].

In this talk, we will introduce a novel Floquet-Bloch transform based method to analyse the radiation condition. Based on the F-B transform, we decompose the scattered field into a Herglotz wave function and a fast decaying function. By analysing the radiation condition for the Herglotz wave function, we can show that $\frac{\partial u}{\partial r} - \imath k u = O\left(r^{-3/2}\right)$.

References

[1] S. N. Chandler-Wilde and P. Monk, Existence, uniqueness and variational methods for scattering by unbounded rough surfaces, *SIAM J. Math. Anal.* **37** (2005), pp. 598 – 618.

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MS23 Tuesday, 12:30–12:55, SR 1.067

On an asympotic far field expansion for 2-d open waveguides

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Different radiation conditions have been proposed for open waveguides and open periodic waveguides where existence and uniqueness have been proved. They consider the guides modes while filtering out the incoming guided modes. Recently, Kirsch showed the decay property of the radiative part of the solution in the direction of the waveguide.

Based on this and using the principles of the method of matched asymptotic expansions we present in this talk asymptotic expansions of the radiative part above and below a layered 2-d waveguide of wavenumber k and inside the waveguide:

$$u(r,\theta) = \sum\nolimits_{j\in\mathbb{N}_0} \Theta_0^{(j)}(\theta) r^{-\frac{1}{2}-j} \mathrm{e}^{\mathrm{i}kr}, \tag{1}$$

$$u(x_1,x_2) = \sum\nolimits_{j\in\mathbb{N}_0} \Theta_1^{(j)}(x_2) x_1^{-\frac{1}{2}-j} \mathrm{e}^{\mathrm{i}kx_1}. \tag{2}$$

References

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Contributed Talks

Contributed Talk **Monday, 11:45–12:10, SR 3.069**

Model-based Super-resolution: Towards a Unified Framework for Super-resolution

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In mathematics, a super-resolution problem can be formulated as acquiring high-frequency data from low-frequency measurements. This extrapolation problem in the frequency domain is well-known to be unstable. We propose the model-based super-resolution framework (Model-SR) to address this ill-posedness. Within this framework, we can recover the signal by solving a nonlinear least square problem and achieve the super-resolution. Theoretically, the resolution-enhancing map is proved to have Lipschitz continuity under mild conditions, leading to a stable solution to the super-resolution problem. We apply the general theory to three concrete models and give the stability estimates for each model. Numerical experiments are conducted to show the super-resolution behavior of the proposed framework. The model-based mathematical framework can be extended to problems with similar structures.

References

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Contributed Talk **Monday, 12:15–12:40, SR 3.069**

Enhanced interpretability of seismic data via multiscale mollifier decorrelation

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We present a multiscale post-processing method in exploration. Based on a physically relevant mollifier technique involving the elasto-oscillatory Cauchy–Navier equation, we mathematically describe the extractable information within 3D geological models obtained by migration as is commonly used for geophysical exploration purposes. More explicitly, the developed multiscale approach extracts and visualizes structural features inherently available in signature bands of certain geological formations such as aquifers, salt domes etc. by specifying suitable wavelet bands.

References

[1] C. Blick and S. Eberle, A survey on multiscale mollifier decorrelation of seismic data, *Int J Geomath* **12**, 16 (2021)

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Contributed Talk **Monday, 15:00–15:25, SR 3.069**

Oblique shock polars for general equations of state

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Shock polars describe the possible post-shock states, in particular velocities, for fixed pre-shock state while the shock angle is varied. For polytropic full Euler flow it is well-known that there is a unique "critical" shock with maximal velocity deflection, with subsonic post-shock state, which is the "standard" case. The talk will discuss hard results as well as numerics for more general equations of state and other pde, including isentropic Euler with the shallow water system as special case, compressible potential flow, ideal non-polytropic equations of state, and various forms of the van der Waals equation. Generally convex ideal gas eos and its simplified forms have "standard" polars, while counterexamples can be given in the non-ideal cases.

Contributed Talk **Monday, 15:30–15:55, SR 3.069**

Propagation of Dispersive Shock Waves in Hall-Magnetohydrodynamics Flows

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We investigate the dynamics of a DSW (Dispersive Shock Wave) within the framework in a Hall-Magnetohydrodyanmics model for van der Waals fluids. An asymptotic method is used to obtain the KdV-type evolution equation characterized by a convex flux function and includes third-order derivatives to account for dispersion effects; where the nonlinearity and the Hall effect appear through quadratic flux function and dispersive term, respectively. The van der Waals parameters a and b influence the nonlinearity and dispersive terms. The interplay between nonlinearity and the Hall effect leads to the emergence of a dispersive shock wave, which appears as the solution to the initial value problem associated with the evolution equation. We focus on the impact of nonlinearity and Hall parameter on various aspects of DSW, such as its formation, the amplitude of the initial profile, and the width and wavelength of the resulting oscillatory waves. It has been observed that an increase in the nonlinearity parameter leads to a more intense oscillatory behavior and results in an early appearance of DSW.

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Contributed Talk	**Monday, 16:30–16:55, SR 3.069**

Robust fully discrete error bounds for the Kuznetsov equation in the inviscid limit

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We consider the Kuznetsov equation, a classical wave model of nonlinear acoustics, on a bounded domain $\Omega \subset \mathbb{R}^d$, $d = 2, 3$,

$$(1 + \kappa\partial_t u)\partial_t^2 u - c^2\Delta u - \beta\Delta\partial_t u + \ell\nabla u \cdot \nabla\partial_t u = f. \tag{1}$$

In the context of nonlinear acoustics, the function u represents the acoustic velocity potential, $c > 0$ denotes the speed of sound in the medium, and the damping parameter β is the sound diffusivity. When its strong damping vanishes, it undergoes a singular behavior change, switching from a parabolic-like to a hyperbolic quasilinear evolution. In this talk, we discuss its finite element discretization as well as a semi-implicit fully discrete approximation that are robust with respect to the vanishing damping parameter and further include numerical examples which confirm our theoretical findings.

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Contributed Talk	**Monday, 17:00–17:25, SR 3.069**

A Posteriori Error Estimates for the Wave Equation with Mesh Change in the Leapfrog Method

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Adaptive mesh refinement is key for the efficient simulation of wave phenomena in complex geometry. In contrast to elliptic and parabolic problems, a posteriori error estimation is less developed for wave equations and typically leads to implicit time integration. Georgoulis et al. derived an a posteriori error estimate for the leapfrog (LF) method, arguably the most popular two-step explicit time integration method for the wave equation. Local mesh refinement, however, severely constrains the time-step of any explicit time-stepping method due to the CFL stability condition governed by the smallest elements in the mesh. LF based local time-stepping (LTS) methods overcome that bottleneck without sacrificing explicitness by taking smaller time-steps only where needed. Here we derive fully discrete a posteriori error estimates for the wave equation when integrated with a LF-based LTS method, incorporating thus adaptivity into explicit time integration with mesh change in time while retaining efficiency.

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Contributed Talk Monday, 17:30–17:55, SR 3.069

Local time-stepping methods for Friedrichs' systems

Marlis Hochbruck[1], **Malik Scheifinger**[1,*]

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In this talk, we address the full discretization of Friedrichs' systems with a two-field structure, such as Maxwell's equations or the acoustic wave equation in div-grad form, cf. [3]. We focus on a discontinuous Galerkin space discretization, applied to a locally refined mesh or a region with large material parameters. This results in a stiff system of ordinary differential equations, where the stiffness is mainly caused by a small portion of the spatial mesh. When using explicit time integration schemes, the time step size is heavily restricted by a few spatial elements, leading to a loss of efficiency. To address this, we propose and analyze a general leapfrog-based scheme which is motivated by [2]. The new local time-stepping method filters the stiff part of the system which weakens the CFL condition of the leapfrog scheme significantly with only a small increase in computational cost. A particular instance of this is the locally implicit method, proposed and analyzed in [1].

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Contributed Talk Monday, 18:00–18:25, SR 3.069

Numerical approximation of discontinuous solutions of the semilinear wave equation

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In this talk, I will present a high-frequency recovery-based fully discrete low-regularity integrator for approximating rough and potentially discontinuous solutions of the semilinear wave equation. The proposed method, with high-frequency recovery techniques, can capture the discontinuities of the solutions correctly without spurious oscillations and approximate rough and discontinuous solutions with a higher convergence rate than pre-existing methods. Rigorous analysis is presented for the convergence rates of the proposed method in approximating solutions such that $(u, \partial_t u) \in C([0,T]; H^\gamma \times H^{\gamma-1})$ for $\gamma \in (0,1]$. For discontinuous solutions of bounded variation in one dimension (which allow jump discontinuities), the proposed method is proved to have almost first-order convergence under the step size condition $\tau \sim N^{-1}$, where τ and N denote the time step size and the number of Fourier terms in the space discretization, respectively. Numerical examples are presented in both one and two dimensions to illustrate the advantages of the proposed method in improving the accuracy in approximating rough and discontinuous solutions of the semilinear wave equation. The numerical results are consistent with the theoretical results and show the efficiency of the proposed method.

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Contributed Talk **Tuesday, 11:30–11:55, SR 3.069**

Finite energy traveling waves for nonlinear Schrödinger equations

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In the present talk, we disclose recent results contained in [1], where we studied the existence of nontrivial finite energy travelling waves in the subsonic regime for a class of nonlinear Schrödinger equations, including the Gross-Pitaevskii.

Motivated by the fact that the so-called Jones-Putterman-Roberts programme has deserved a lot of attention in the literature, we consider more general growth assumptions at infinity, in the spirit of Berestycki-Lions, regarding the nonlinearity, compared to those already known, and we provide a direct and simpler variational approach based on a new Sobolev-type inequality involving the momentum.

References

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Contributed Talk **Tuesday, 12:00–12:25, SR 3.069**

Special wave forms for a generalized semilinear wave equation

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We study the generalized semilinear wave equation

$$V(x)u_{tt} - d(t)M(x,\partial_x)u - V(x)|u|^{p-1}u = 0 \quad \text{for} \quad (x,t) \in \mathbb{R}^N \times \mathbb{R}$$

where M is elliptic and d is a positive potential. Our goal is to construct solutions which are localized in space and/or time by means of variational methods. We present our approach with its main difficulties and discuss suitable examples for M and d. This is joint work with Sebastian Ohrem and Wolfgang Reichel.

Contributed Talk | Tuesday, 12:30–12:55, SR 3.069

Endpoint Strichartz estimates and small data scattering for cubic Dirac equations on a curved background

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This work is devoted to the study of global behaviour of low regularity solutions to cubic Dirac equation on a $(1+3)$-dimensional curved space-time. We introduce a specific pseudo-differential operator to reformulate the Dirac equation into the half-Klein-Gordon equation as one can obtain in the flat space setting. Then we establish the endpoint Strichartz estimates for the half-Klein-Gordon equations, with a weak asymptotic flatness assumption on the metric. As a direct application of the Strichartz estimates, we are able to prove the global well-posedness and scattering of the cubic Dirac equation with small data in H^s, $s > 1$. This is joint with Sebastian Herr.

Contributed Talk | Wednesday, 11:30–11:55, SR -1.025

Low Regularity Time Integration of the Boussinesq Equation

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We investigate a filtered Lie splitting scheme for the time integration of the ‘good’ Boussinesq equation with initial data in the Sobolev space $H^s(-\pi,\pi)$, $s > 0$. Our focus is on very small values of s. The main challenge with such low regularity initial data is that for $s \le 1/2$, the stability of the scheme cannot be analyzed using standard Sobolev techniques due to the invalidity of the bilinear estimate for these values of s. To address this problem, we reformulate the Boussinesq equation as a first-order evolution equation. This reformulation shows similarities to a nonlinear Schrödinger equation, enabling the application of the recently introduced framework of discrete Bourgain spaces [1,2] and allowing us to establish convergence [3]. In particular, for τ, the step size of the method, we prove convergence of order $\tau^{s/2}$ in $L^2(-\pi,\pi)$ for $0 < s \le 2$. These analytical results are supported by numerical experiments.

This is joint work with Lun Ji, Hang Li, and Chunmei Su.

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Contributed Talk Wednesday, 12:00–12:25, SR -1.025

Error analysis of splitting methods for semilinear wave equations with finite-energy solutions

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We study time integration schemes for H^1-solutions to the energy-(sub)critical semilinear wave equation on the full space $\mathbb{R}^3$ and the torus $\mathbb{T}^3$. We show first-order convergence in L^2 for the Lie splitting. To our knowledge this includes the first error analysis performed for scaling-critical dispersive problems. In the case of a cubic nonlinearity, we show almost second-order convergence for the Strang splitting. Our approach is based on discrete-time Strichartz estimates, including one (with a logarithmic correction) for the case of the forbidden endpoint. Our schemes and the Strichartz estimates contain frequency cut-offs. By similar methods, related results for the semilinear Schrödinger equation have been obtained by Katharina Schratz and co-authors, for instance. This is partly joint work with Roland Schnaubelt [1].

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Contributed Talk Wednesday, 12:30–12:55, SR -1.025

Time integration method for wave propagation with spatio-temporal oscillations

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For nonlinear Friedrichs systems with solutions that oscillate rapidly in space and time, it is a major challenge to compute reasonable approximations efficiently. We tackle this problem by a fusion of analytical and numerical approximation techniques. First, we replace the original PDE system with a fine-tuned modification of the slowly varying envelope approximation. In the second step, we devise an efficient and uniformly accurate time integration method tailored to our novel modified PDE system. Central to this is the analysis of interactions between oscillatory and non-oscillatory parts of the solution.

Contributed Talk	Wednesday, 11:30–11:55, SR 3.069

Impact of multiple porous rings on water wave scattering by a bottom-mounted wind turbine

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Offshore wind farms are crucial for generating renewable marine energy, requiring well-designed wind turbines for optimal performance, longevity, and cost efficiency. Monopiles in these farms often face significant hydrodynamic forces. Porous structures can enhance the hydrodynamic performance of monopiles by controlling wave activity. Wu and Chwang [1] analyzed wave scattering by a vertical cylinder with a submerged horizontal ring-type porous plate, and inferred reduced wave impact. Building on this, we extend this work to study wave action on a bottom-mounted impermeable vertical cylinder fitted with multiple horizontal porous rings below the free surface, using linear water wave theory and the matched eigenfunction expansion method. Numerical results examine and estabish the effects of porosity, ring dimensions, and water depth on wave forces.

References

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Contributed Talk	Wednesday, 12:00–12:25, SR 3.069

Wave scattering by an ice-floe in the presence of a thick porous sea-bed

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Flexural gravity waves are of great interest to researchers because they play an important role in sea ice dynamics, influencing its breakage, deformation, and overall behaviour in polar regions. Precise models of these interactions are required for predicting sea ice conditions and assessing the impact of climate change on polar ecosystems. Langhorne et al. [1] studied sea ice breaking and floe size dispersion and found that the endurance limit of sea ice is around 60% of its flexural strength. In natural marine habitats, the sea-bed is more correctly described as a thick porous medium. Understanding of water wave interaction with porous sea-beds is critical for developing efficient coastal protection structures, such as breakwaters and seawalls, to mitigate the storm surges and increasing sea levels. The work in [1] is extended to study wave scattering in an ice-floe with the consideration of a thick porous sea-bed. Numerical results establish the effects of the sea-bed and plate parameters on the scattering coefficients, the wave forces and the plate deflection.

References

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Contributed Talk Wednesday, 12:30–12:55, SR 3.069

Higher-order rogue waves for DNLS equation

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Rogue waves have been studied for decades because of their features, such as unpredictability and their huge amplitude [1,2,3]. In this study, we numerically solve discrete nonlinear Schrödinger equation under periodic boundary conditions. We study the amplitude of rogue waves according to the proper choice of initial conditions. We predict higher-order discrete rogue waves.

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Contributed Talk Wednesday, 15:00–15:25, SR 3.069

Semi-analytical solution of (2+1)-dimensional dispersive long wave equations by means of Homotopy Analysis Method

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Abstract

This paper employs the Homotopy Analysis Method (HAM) to obtain approximate solutions to the (2+1)-dimensional dispersive long wave equation. This equation represents a variety of physical phenomena, including ocean waves, atmospheric waves, and plasma physics. The HAM is used to obtain semi-analytical solutions while overcoming the limits of existing methods. Rigorous analysis ensures that the solutions converge. The numerical results illustrate the HAM approach's accuracy and efficiency. Comparative research with established methodologies support the conclusions.The results show that HAM is efficient at solving highly nonlinear systems, making it a useful tool for dealing with a wide range of nonlinear PDEs. Convergence of the HAM-based solution is demonstrated via the squared residual error approach. It is shown that there is a strong match between the HAM-based technique and the exact solution to the problems.

Keywords: (2+1)-dimensional dispersive long wave equation, Homotopy Analysis Method, Non-linear PDEs, Semi-analytical solutions.

Contributed Talk	Wednesday, 15:30–15:55, SR 3.069

Semi-analytical solution of (*1+1*)-dimensional Kaup system by means of Homotopy Analysis Method

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Abstract

This study presents a semi-analytical solution to the (1+1)-dimensional Kaup system, a nonlinear partial differential equation modeling shallow water waves and ion-acoustic waves in plasmas. The Homotopy Analysis Method (HAM) is employed to derive a convergent series solution. The effects of physical parameters on wave dynamics are thoroughly investigated. The results show that HAM provides a reliable and efficient approach for solving the Kaup system, overcoming limitations of traditional methods. Convergence of the HAM-based solution is demonstrated via the squared residual error approach. It is shown that there is a strong match between the HAM-based technique and the exact solution to the problem.

Keywords: Homotopy Analysis Method, Kaup System, Semi-analytical Solution, Nonlinear Partial Differential Equations, Shallow Water Waves.

Contributed Talk	Thursday, 11:30–11:55, SR 3.069

Bulk-boundary Correspondence in Topological Electrodynamics: From 1D to 1.5D to 2D

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A cornerstone of topological electrodynamics is the bulk-boundary correspondence principle (b-bc): the presence of interface modes between two periodic structures depends on discrete topological invariants (Zak phases in 1D and Chern numbers in higher dimensions) of the respective Bloch bands.

In physics literature, analyses are typically performed only for Bloch modes in the bulk; thus, conclusions about the boundary behavior of fields are reached after ignoring this behavior in the first place – an illogical loop. Moreover, a puzzling feature of b-bc is that the properties of *evanescent* modes in a band gap somehow depend on the properties of *propagating modes* at completely different frequencies.

The paper demystifies this connection. A complete analysis and results are available for problems in 1D and "1.5D" (two-component fields but material parameters depending on one coordinate only). A key observation is the monotonicity of Bloch impedance as a function of frequency in any bandgap. It is shown, furthermore, that for media with frequency dispersion the b-bc is closely related to the positivity of electromagnetic energy density.

The 2D problem is qualitatively more complex. The Bloch impedance turns into an infinite-dimensional Neumann-to-Dirichlet (NtD) operator. As a generalization of the 1D case, the NtD is proved to be monotone within any bandgap for any physically realizable media with positive energy density. Several numerical examples are presented.

Contributed Talk **Thursday, 12:00–12:25, SR 3.069**

Inverse-designed time-varying nanostructures

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Time-varying nanostructures are those photonic structures whose material properties are modulated periodically as a function of time. Here, the frequency of modulation is comparable to the oscillation frequency of light. The temporal modulation unlocks additional degrees of freedom to control the flow of light in a reconfigurable manner. However, the additional degrees of freedom create a vast design space which makes the identification of optimal devices with predefined functionalities difficult. Photonic inverse design offers a computationally efficient solution to this issue.

Here, we present a differentiable transition (T-) matrix-based framework to inverse design the time-varying nanostructures [1]. Depending on the choice of objective function, various functionalities can be realized using the framework. In particular, we exemplify our approach by realizing the anomalous Drexhage effect using time-varying spheres, and asymmetric transmission of light using time-varying metasurfaces by appropriately designing the time-varying material properties.

References

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Contributed Talk **Thursday, 12:30–12:55, SR 3.069**

Topologically protected modes in systems with dispersion and damping

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The work presented extends the theory of topological protection to systems with dispersion and damping. We prove the existence of localised interface modes using the monotonicity of impedance functions, Rouché's theorem and the geometric symmetries of the material. We consider time-harmonic waves in one-dimensional systems. Finally, we show that, when such modes exist, they benefit from enhanced robustness with respect to imperfections. This work can be found in [1,2].

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Contributed Talk	Thursday, 15:00–15:25, SR 3.069

Reflection-Transmission Coefficients of SH Waves Across Thin-Walled Spring-Membrane Strain Gradient Interface

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The current investigation focuses on the reflection and refraction phenomena of SH waves as they encounter two distinct non-ideal interfaces positioned between two half-spaces. The initial interface consists of a spring layer positioned between two membranes, while the second interface is comprised of a thin membrane based on strain gradient principles. Notably, the investigation is done for two cases, namely, the Spring Membrane Strain Gradient (SMSG) and the Spring Membrane Surface Elasticity (SMSE). Analytical derivations have been performed for the reflection and transmission coefficients, as well as the phase shifts associated with both interfaces in the context of reflection and refraction phenomena. The case SMSE and SMSG has the potential to transition into alternative interface models when specific limits of the interface parameters are applied, which has been discussed in detail as particular cases. Moreover, to enhance the understanding of the diverse parameters, we have graphically represented the amplitude ratios and phase shifts mentioned above. When examining the behaviour of reflection and refraction coefficients at different angle of incidence, the presence of one versus two interfaces introduces additional complexity. As the angle of incidence increases the interaction between the wave and the material interfaces changes. This complex interplay is crucial for designing materials and structures in application like acoustic insulation, optical devices, and advanced engineering materials.

Contributed Talk	Thursday, 15:30–15:55, SR 3.069

Characteristics of wave propagation in Pre-stressed Viscoelastic Timoshenko Nanobeams with Surface Stress and Magnetic Field Influences

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The current investigation explores the behavior of pre-stressed viscoelastic Timoshenko nanobeams under the influence of surface effects and a longitudinal magnetic field. Utilizing a modified version of non-local strain gradient theory through the Kelvin-Voigt viscoelastic model, a closed-form dispersion relation using a suitable analytical approach has been derived. To account for surface stresses, Gurtin-Murdouch surface elasticity theory has been employed. Additionally, the study delves into the impact of a longitudinal magnetic field on a single-walled carbon nanotube, considering Lorentz magnetic forces. The validity of the findings is established by deriving results in the absence of surface effects and magnetic fields, aligning well with existing literature. The investigation indicates that pre-stress has marginal effects on flexural and shear waves, while surface effects, magnetic fields, non-locality, characteristic length, and nanotube diameter significantly influence the phase velocity. Additionally, the Threshold velocity and blocking diameter are discussed for the model.

Contributed Talk **Thursday, 16:30–16:55, SR 3.069**

Polynomial density interpolation for fast, high-order numerical evaluation of volume potentials in wave scattering

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We present a novel class of high-order methods for efficiently evaluating volume potentials (VPs) over complex geometries. Inspired by the Density Interpolation Method (DIM) for boundary integral operators, this approach uses Green's third identity and local polynomial interpolation to recast a VP as a combination of surface-layer potentials and a regularized volume integral. The layer potentials are efficiently evaluated using existing methods (e.g., DIM), while the regularized volume integral is computed accurately using simple quadrature rules over structured or unstructured decompositions, without special treatment for kernel singularities. This flexible methodology is compatible with fast algorithms like the Fast Multipole Method and $\mathcal{H}$-matrices, achieving linearithmic computational complexity. We demonstrate its effectiveness on the Nyström discretization of the Lippmann-Schwinger volume integral equation for Helmholtz scattering problems in piecewise-smooth variable media.

References

[1] T. G. Anderson, M. Bonnet, L. M. Faria, and C. Pérez-Arancibia, Fast, high-order numerical evaluation of volume potentials via polynomial density interpolation, *Journal of Computational Physics* **511**, 113091.

Contributed Talk **Thursday, 17:00–17:25, SR 3.069**

First-kind Galerkin BEM for the Hodge–Helmholtz equation

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We are interested in exterior boundary value problems (BVP) for the Euclidean Hodge-Helmholtz operator, which is closely related to time-harmonic Maxwell equations. To solve these BVPs, we consider the corresponding first-kind boundary integral equations, which were derived and analyzed in [2]. One can establish their unique solvability, however, the situation changes when the wavenumber κ is zero. Then, the related sesquilinear forms feature nontrivial kernels. In this talk, we pursue the Galerkin discretization of the variational formulations in [2] and we provide numerical experiments using Bempp [1]. We validate our implementation using a new Calderón residual technique. Then, we compare the obtained eigenvalues for $\kappa = 0$ with those found in [3] and also present the spectra for small wave numbers κ. Finally, we discuss the numerical behavior when κ is not zero but is in the order of machine precision.

References

[1] T. Betcke and M. W. Scroggs, Bempp-cl: A fast Python based just-in-time compiling boundary element library, *Journal of Open Source Software* **6(59)** (2021).

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[3] X. Claeys and R. Hiptmair, First-kind Galerkin boundary element methods for the Hodge-Laplacian in three dimensions, *Mathematical Methods in the Applied Sciences* **43(8)** (2020), pp. 4974–4994.

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